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GROUNDING AND LOGICAL BASING PERMISSIONS

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Abstract. The relation between logic and rationality has recently re-emerged as an important topic of discussion. Following the ideas of Broome [1999] and MacFarlane [2004], the debate focused on providing rational requirements, which work as bridges between logic and epistemic norms. However, as Broome [2014] and Way [2011] observed, the usual requirements cannot capture some important aspects of rationality, such as how one can rationally believe something on the basis of believing something else. Broome [2014] proposed a few additional principles ("basing permissions") for this purpose. In this paper I develop a more systematic family of basing permissions using the recent notion of grounding (Fine [2012], Correia [2014]). In particular, I claim that if Γ (logically) grounds A, and you believe Γ, then rationality permits you to believe A on the basis of believing Γ.

Keywords: logic and rationality, basing permissions, rational requirements, bridge principles, grounding.

1. INTRODUCTION

The relation between logic and rationality is one of the most discussed topics in philosophy of logic. Many authors in the analytic tradition presupposed that in order to be rational, agents need to follow the rules of logic.[1] The discussion reappeared after Broome's [1999] work, which focused on the precise formulation of the requirements of rationality. Later on, authors such as MacFarlane [2004], Field [2009], and Broome [2014], proposed a number of positive theories about the logical constraints on rationality.

Most authors agree that the relation between logic and rationality should be characterized by some bridge principles. It is hard to give a sufficiently neutral and general presentation of the bridge principles; for simplicity, I will mostly follow Broome's [2014] version. In Broome's presentation, the principles involve the operator "rationality requires that…"; so we can call them "rational requirements". An important aspect of these principles is the scope of the rationality ope-

[1] For a detailed analysis about the relation between logic and rationality in the philosophical tradition, particularly in Kant and Frege, see MacFarlane [2002].

rator, which can be wide or narrow. The standard narrow scope requirement for logic has this form:

> (**NS VALIDITY**) If $\Gamma \vDash A$ and you believe Γ, then rationality requires you to believe A.

On the other hand, the standard wide scope requirement for logic can be expressed in this way:

> (**WS VALIDITY**) If $\Gamma \vDash A$, then rationality requires you not to believe some sentence of Γ, or to believe A.

The authors I mentioned above argue for variations of WS VALIDITY, with some modifications regarding the operator, the difficulty of inferences, and the relevance of the sentences. The precise formulation of these requirements is a matter of dispute, and the correct way of introducing the aspects of difficulty and relevance can be discussed largely.

Anyway, in this paper I deal with a specific aspect of logical rationality. Sometimes we appeal to rational requirements to hold one attitude on the basis of having other attitudes. In particular, one can *believe* something on the basis of believing something else. This is the general problem of *basing* [Broome 2014], which is central for theories of rationality. This paper will address the logical kind of basing: the question is which beliefs we can adopt in virtue of holding other beliefs that are logically connected.[2] Just to illustrate, the following two examples seem perfectly rational:

> **Example 1:** you believe that the moon is blue or white on the basis of believing that the moon is white.
>
> **Example 2:** you believe that some Dalmatians are crazy on the basis of believing that your dog is Dalmatian and crazy.

It is important to notice that in the examples above, you arrive at contingent (and not tautological) sentences. In other words, the examples illustrate the following fact:

> **(Contingent reasoning)** For some contingent sentences A and some sets of sentences Γ which imply A, rationality permits you to believe A on the basis of believing Γ.

Unfortunately, neither WS VALIDITY nor NS VALIDITY can capture Contingent

[2] It is worth remarking that epistemic basing principles can exceed logical basing principles. For example: "rationality allows you to believe *p* on the basis of believing that you have evidence for *p*" may be a non-logical epistemic basing principle.

reasoning. For, suppose that you believe A, and you believe A ∨ B on the basis of believing A. WS VALIDITY cannot explain this process: this requirement just asks you either not to believe A or to believe A ∨ B, but it does not say anything about the possibility of believing A ∨ B on the basis of believing A.[3] One might feel tempted to supplement WS VALIDITY with the following principle:

> (**DISJUNCTIVE REQUIREMENT**) If rationality requires you to either lack the attitude A or to have the attitude B, then it permits you to have the attitude B on the basis of having the attitude A.

If DISJUNCTIVE REQUIREMENT is accepted, then WS VALIDITY (which required you either not to believe A or to believe A ∨ B) allows you to believe A ∨ B on the basis of believing A. Indeed, the following principle can be obtained:

> (**VALIDITY PERMISSION**) If Γ implies A, then rationality permits you to believe A on the basis of believing Γ.

Nevertheless, VALIDITY PERMISSION is affected by a problem which appeared in different versions in Broome [1999, 2014] and Kolodny [2005] under the name of *bootstrapping*. Given the reflexivity of logic (i.e. A implies A), VALIDITY PERMISSION permits you to believe A on the basis of believing A, for every A.[4] And this is clearly absurd, as the following example illustrates:

> **Example 3:** you believe that the moon is white or blue on the basis of believing that the moon is white or blue.

Moreover, VALIDITY PERMISSION permits you to make many kinds of relevance fallacies. For example, it permits you to believe every sentence on the basis of believing any inconsistent set of sentences; and it permits you to believe every tautology on the basis of believing any other sentence.

Therefore, it is better to reject VALIDITY PERMISSION (which also involves the rejection of DISJUNCTIVE REQUIREMENT). But in this way, the wide scope requirements do not permit you to base specific beliefs on other beliefs; they just forbid you to adopt a particular set of beliefs (for example, believing A and not believing A ∨ B).

Observe that this is also a problem for NS VALIDITY. Even though NS VALIDITY has narrow scope, it is not a principle of basing. A natural way of establishing a connection would be to transform instances of NS VALIDITY, such

[3] Kolodny [2005] expresses the point differently: a wide scope principle just forbids to adopt a set of attitudes; while an actual rational requirement should describe how to "resolve" this conflicting set, i.e. how to reason from the content of an attitude to revising an incompatible attitude.

[4] In some cases, one might rationally believe A on the basis of believing A. For example, if you believe "I have a belief" on the basis of believing "I have a belief". But this cannot be generalized.

as "if you believe A, rationality requires you to believe A ∨ B" into instances of VALIDITY PERMISSION, such as "if you believe A, rationality permits you to believe A ∨ B on the basis of believing A". But then, VALIDITY PERMISION can be derived, and the bootstrapping problem reappears.

In this paper, I will offer some supplementary requirements which can be used as *basing permissions*, i.e. can help to determine which are the rationally permitted ways of basing a belief on other beliefs. The strategy of trying to find supplementary basing permissions was suggested by other authors, such as Way or Broome.[5] My aim in this paper is to provide an interesting, although non-exhaustive, set of basing permissions for logical rationality.

2. GROUNDING

In the last section, I argued against VALIDITY PERMISSION (i.e. the idea that if Γ implies A, then rationality permits you to believe A on the basis of believing Γ). Nevertheless, my proposal is similar to that idea. Remember that the main problem of VALIDITY PERMISSION was the bootstrapping problem, i.e. it allows you to believe A on the basis of believing A. In order to avoid that problem, my strategy is to adopt an irreflexive consequence relation, recently developed under the name of *grounding*.

Logical grounding

Grounding is usually conceived as a non-causal explanatory relation between facts. For example, the fact that this ball is round and the fact that it is red ground the fact that it is red and round. The fact that certain action was performed to cause harm (presupposing certain moral theory) grounds the fact that the action was wrong.[6] In most cases in which *p* grounds *q*, we can say that "*q* because *p*": "I am happy and alive because I am happy and I am alive", "this figure is squared because it has four identical sides", etc. This explains why some authors [Schnieder 2011] take the notion of grounding as a metaphysical correlate of the *because* operator. Moreover, grounding has a modal force: when A grounds B, necessarily if A then B. However, grounding cannot be reduced to necessitation. For example,

[5] Way [2011] p. 230; Broome [2014] p. 187–188. Both Broome and Way are wide-scopers who believe in the necessity of providing supplementary basing principles. According to Way ([2011] p. 230), basing principles specify the "conditions under which attitudes can be appropriately formed, held and revised". Nevertheless, he does not suggest particular examples. On the other hand, Broome ([2014] p. 189) claims that the notion of "basing" is a primitive, but he provides some examples of basing permissions, as we will see below.

[6] These examples can be found in Fine [2012] p. 37.

necessarily if it rains, then 6+4=10; but the first truth does not ground the second one.

Schnieder and Correia[7] distinguish at least three kinds of grounding:

– Logical grounding. For example, "2+2=4" grounds "2+2=4 or 2+2=5".
– Conceptual grounding. For example, "2+2=4" grounds "it is true that 2+2=4".
– Metaphysical grounding. For example, "Socrates and Plato exist" grounds "{Socrates, Plato} exists".

We will not discuss in detail the classification of the different types of grounding. But it is important to point out that in this paper I am using the *logical* notion of grounding, which is a proper subset of the relation of (classical) logical consequence. Moreover, and just for methodological reasons, I will take grounding as a relation between sentences (or in any case, structured propositions), not between facts.[8]

We will understand grounding as the notion of (logical) *strict grounding* developed by Fabrice Correia [2014], which has the same operational rules as the notion of *full strict grounding* developed by Kit Fine [2012].[9] Following Fine's terminology, we will use the symbol "<" to express this notion: we say that $\Gamma<A$ whenever Γ grounds A.

Grounding (i.e. logical strict grounding) satisfies three important properties:

1. Irreflexivity (i.e. no sentence A is such that $A<A$).
2. Transitivity (i.e. if $\Gamma<A$ and $A<B$, then $\Gamma<B$).
3. Subclassicality (i.e. if $\Gamma<A$, then Γ logically implies A).[10]

The logic of grounding can be expressed as a natural deduction with subscripts (Anderson, Belnap [1975]; Mares [2004]).[11] In these proof systems, a derivation begins with the enumeration of the premises, each one with a different single-

[7] Schnieder, Correia [2012] p. 21.

[8] According to Correia and Schnieder ([2012] p. 6), this was Bolzano's position.

[9] The systems of Fine and Correia include the same operational rules, but different structural rules. In particular, Fine accepts Amalgamation: if Γ_1 grounds A, …, and Γ_n grounds A, then $\Gamma_1 \cup \ldots \cup \Gamma_n$ grounds A.

[10] Fine [2012] develops other notions of grounding which are reflexive ("weak grounding"), non--transitive ("immediate grounding") or not subclassical ("partial grounding"). However, we are not concerned about these notions here.

[11] This is not the way in which Fine [2012] and Correia [2014] present their systems. Fine uses a sequent calculus, and Correia appeals to trees. However, the labeled natural deduction calculus is much simpler, and strongly resembles Correia's approach.

ton as a subscript, and one arrives to the conclusion by using inference rules. Every time a formula is obtained by means of a rule, it contains as a subscript the same set as the premise (if it is a one-premise rule) or the union of the subscripts of the premises (if it is a two-premise rule); in this way, the derivation keeps track of which hypotheses are used. For an inference to be valid, every premise must be used in the derivation (this is indeed the purpose of the subscripts). In formal terms, {A1, …, An} < B iff there is a proof of $B_{\{1,\dots n\}}$ which begins with $A1_{\{1\}}, \dots An_{\{n\}}$.

These are the rules for the logic of grounding:

…
m. φ_A
…
n. $\varphi \vee \psi_A$ I∨, *m*

…
m. ψ_A
…
n. $\varphi \vee \psi_A$ I∨, *m*

…
m. φ_A
…
n. $\neg\neg\varphi_A$ I¬¬, *m*

…
m. $\neg\varphi_A$
…
n. $\neg(\varphi \wedge \psi)_A$ I¬∧, *m*

…
m. $\neg\psi_A$
…
n. $\neg(\varphi \wedge \psi)_A$ I¬∧, *m*

…
m. φ_A
…
n. ψ_B
…
o. $\varphi \wedge \psi_{A \cup B}$ I∧, *m*, *n*

…
m. $\neg\varphi_A$
…
n. $\neg\psi_B$
…
o. $\neg(\varphi \vee \psi)_{A \cup B}$ I¬∨, *m*, *n*

We can see now, as an illustration, why {A,B} grounds ¬(A∧C) ∧ ¬¬B:

1.	$\neg A_{\{1\}}$	Premise
2.	$B_{\{2\}}$	Premise
3.	$\neg(A \wedge C)_{\{1\}}$	I¬∧, 1
4.	$\neg\neg B_{\{2\}}$	I¬¬,2
5.	$\neg(A \wedge C) \wedge \neg\neg B_{\{1,2\}}$	I∧,3,4

It is easy to see that grounding is *subclassical*, since all its rules are valid inferences in classical logic. The configuration of the proofs guarantees also that *transitivity* is satisfied. For, if one can derive $B_{\{1,\dots,n\}}$ from $A1_{\{1\}}, \dots, An_{\{n\}}$, and one can derive C_Δ from B_Δ, then it is possible to derive $C_{\{1,\dots,n\}}$ from $A1_{\{1\}}, \dots, An_{\{n\}}$.

The rules above, on the other hand, rule out *reflexivity*: the conclusion of a rule is necessarily more complex (i.e. it includes more connectives) than each of the premises, so A can never ground A. It is worth mentioning that adding complexity from premises to conclusion in a valid argument is not a *sufficient* condition for it to be a case of grounding: for example, $\neg\neg A$ does not ground $A \vee (\neg B \vee \neg C)$. Finally, *monotonicity* (if $\Gamma < A$ then $\Gamma, \Delta < A$) is clearly not satisfied. For example, $\{p\}$ grounds $p \vee q$, but $\{p,r\}$ does not.

In this paper, I will mostly focus on the propositional logic of grounding. However, it is worth mentioning that grounding systems can be extended with rules for *quantifiers*. There are two almost uncontroversial rules:

...	...
$m.\ \neg\varphi(a)$	$m.\ \varphi(a)$
...	...
$n.\ \neg\forall x\varphi(x)$ $\mathrm{I}\neg\forall, m$	$n.\ \exists x\varphi(x)$ $\mathrm{I}\exists, m$

These two rules specify how to introduce a negated universal quantifier, and an asserted existential quantifier. The problem arises with the other two rules that a first-order grounding system should include: the introduction of the (asserted) universal quantifier, and the introduction of the negated existential quantifier. For reasons of space, I will skip this issue here.[12]

3. Basing Principles and Grounding

In the last section I described the logic of grounding. This section will specify the role of grounding in the discussion about rationality. I will argue for the following basing principle:

> (**GROUNDING PERMISSION**) If Γ grounds A, and you believe Γ, then rationality permits you to believe A on the basis of believing Γ.

This principle gives us the two things we asked for. On one hand, unlike WS and NS VALIDITY, it can capture the role of logical requirements in our basing processes. Indeed, it explains the examples 1 and 2 above. On the other hand, given the irreflexivity of grounding, the bootstrapping objection (which affected VALIDITY PERMISSION) is now avoided.[13] In what follows, I will explore some further properties of this principle.

[12] See Fine [2012] for a development of different rules for the quantifiers.

[13] A natural objection to my proposal is that every non-reflexive logic would work as well, so there are no specific reasons to choose a grounding logic. However, there are not many non-reflexive

Factivity

According to the majority of authors, grounding is *factive*: if A grounds B, then both A and B must be true. This is unsurprising, since grounding is an explanatory relation, and factivity is usually taken to be a property of explanation (Hempel [1965]); in other words, in a correct explanation both the *explanans* and the *explanandum* must be true. In a sense, the factivity of grounding would make GROUNDING PERMISSION more plausible: the principle would only allow the agent to believe true sentences.

Anyway, there are also reasons to adopt a non-factive notion of grounding in the basing principles. Sometimes it seems reasonable to adopt a belief on the basis of believing a false sentence. For example, you might believe that there is an odd number that can be divided by two on the basis of believing that 5 is odd and that it can be divided by 2. Even though in this example you adopt a false belief on the basis of another false belief, it seems that the use of the basing permission is correct. Moreover, some authors consider that sentences about taste or morality can be neither true nor false (cf. Gibbard [1990]). However, one can still use those sentences to infer new beliefs. For example, even if "mayonnaise is tasty" had no truth value, one could still believe "mayonnaise is tasty or interesting" on the basis of believing "mayonnaise is tasty".

Therefore, if these epistemic practices are to be captured, GROUNDING PERMISSION should involve a non-factive notion of grounding. Actually, adopting a non-factive notion is not problematic. Even though the tradition of Bolzano has taken grounding as factive, the notion of grounding we use here (Correia's "strict grounding") is non-factive. For example, according to our notion of grounding, $\{p,\neg p\}$ grounds $p \wedge \neg p$, even when the premises cannot be true at the same time.

Hyper-intensionality

One particularity of the basing principles is their hyper-intensionality. For example, the following sentence is true, according to my view:

(*) If you believe A, then rationality permits you to believe **¬¬A** on the basis of believing A.

logics in the market, and the few that were developed are not useful for my purpose. The "non--reflexive logics" that were developed in philosophy of physics (Bueno, daCosta [1999]) are just logics which do not imply x=x. On the other hand, the relevant logic S, developed by Martin and Meyer [1982], is non-reflexive in the sense that it does not imply A → A. However, in order to adopt this logic to the problem I am interested in, I first need to identify the conditional with the notion of validity. Besides, S is too weak: it does not include other connectives apart from the conditional.

If the substitution of classical equivalents under the scope of the operator were allowed, this sentence would also be true:

(**) If you believe A, then rationality permits you to believe **A** on the basis of believing A.

However, (**) is false. This counts in favor of GROUNDING PERMISSION, since grounding is a hyper-intensional concept. More precisely, the rules of grounding are not closed under substitution of classical equivalents: it might be the case that A<B but A′ does not ground B′, where A′ and B′ are classically equivalent to A and B respectively. Following the previous example, A grounds ¬¬A, but of course A does not ground A. Given this feature, GROUNDING PERMISSION validates (*) but does not validate (**).

The hyper-intensionality of grounding illuminates an aspect of my proposal that could be deemed problematic. Even though the bootstrapping objection is avoided, there is a related problem, which may be called "weak bootstrapping":

(**Weak bootstrapping**) Assume that GROUNDING PERMISSION is true. Then, if you believe A, rationality permits you to believe some logically equivalent sentences, such as ¬¬A or A ∨ A, on the basis of believing A.

Weak bootstrapping is an interesting argument against GROUNDING PERMISSION. However, the previous cases are not necessarily counter-intuitive: for example, believing A may allow you to believe ¬¬A. Indeed, one can say "I believe ¬¬A because I believe A". Thus, complexity is not always a matter of content but also a matter of *form*. In this sense, the hyper-intensionality of the notion of grounding clarifies some ways in which complex beliefs (in the sense of content or form) can be based on simpler beliefs.

Relevance

A positive feature of GROUNDING PERMISSION is that it respects some principles of relevance. Many paraconsistent philosophers have this worry[14]:

(**Anti-explosion**) If the *ex falso* rule is valid, then if you believe an inconsistent set Γ, rationality permits you to believe any sentence on the basis of believing Γ. However, we sometimes believe an inconsistent set Γ, and it is not reasonable to claim that in those cases rationality allows us to believe every sentence on the basis of believing Γ.

[14] Cf. Meyer [1971] p. 814. Steinberger [forthcoming] develops a detailed analysis of this argument.

This argument might certainly be used against VALIDITY PERMISSION. But it does not affect GROUNDING PERMISSION, since in the logic of grounding a contradiction does not ground every sentence.

Indeed, the notion of (propositional) grounding is relevant:

Theorem 1 If $\Gamma<A$, then Γ and A share some propositional letters (i.e. there are propositional letters in common between some formulas of Γ and A).

Proof Assume that $\Gamma<A$. Then, there is a derivation of A from Γ, using the rules of grounding. All these rules are valid in *FDE* (this can be easily checked using truth tables). Therefore $\Gamma\vDash_{FDE}A$. Given that *FDE* satisfies the letter sharing property (Priest [2008]), Γ and A share some propositional letters.

One might prove a similar argument for *verum ex quodlibet*. If VALIDITY PERMISSION is adopted, then if you believe an arbitrary proposition A, rationality permits you to believe $B \vee \neg B$ (or any classical tautology) on the basis of believing A. This is not compatible with our basing processes: one cannot say "I believe $B \vee \neg B$ because I believe A". It is easy to notice that GROUNDING PERMISSION avoids this problem, for not every sentence grounds every tautology.

The relevance of the notion of (propositional) grounding can be extended with another result. In this logic, every propositional letter which appears in the premises must also appear in the conclusion:

Theorem 2 If $\Gamma<A$, then every propositional letter which appears in a formula of Γ, also appears in A.

Proof Every rule of grounding has the following property: each of the letters which appear in the premises, also appear in the conclusion. If $\Gamma<A$, then A can be derived from Γ using every premise Γ. Therefore, A includes every propositional letter of the premises Γ.

One can interpret this atom-containment property as saying that the information contained in the premises is also contained in the conclusion. This property has received some attention recently.[15]

[15] Ciuni and Carrara [forthcoming] observe that *PWK* (a weak-Kleene matrix where the intermediate value is designated), a logic developed by Hallden and Bochvar, satisfies a weaker version of atom-containment. This weaker version says that there is a *subset* Γ' of the premises Γ such that every atom in Γ' is contained in the conclusion. It is easy to show (using truth-tables) that grounding is a sublogic of *PWK*.

Exhaustivity

I argued above that GROUNDING PERMISSION is a logical rational requirement, and particularly a basing principle. But it is not necessarily the only one: other basing principles may also be true.

First, the notion of grounding that I used focuses in some connectives but ignores others, such as the conditional. It might be reasonable to adopt other basing principles with respect to these connectives, such as MODUS PONENS PERMISSION:[16] if you believe that A and you believe that $A \rightarrow B$, then rationality permits you to believe B on the basis of believing A and $A \rightarrow B$.[17]

These additional requirements are not for free, and they involve some concessions. In the particular case of MODUS PONENS PERMISSION, the requirement produces a kind of bootstrapping situation: if you believe A and $A \rightarrow A$, then you can believe A on the basis of believing A and $A \rightarrow A$. One might try to solve this problem, for example, by asking A and B to be different sentences. In this case, irreflexivity is not affected. But the new principle does not satisfy a property which GROUNDING PERMISSION satisfied:

> (**Complexity**) Rationality just allows you to believe more complex beliefs on the basis of believing simpler beliefs; i.e. if rationality permits you to believe A on the basis of believing Γ, then no sentence in Γ can have more logical connectives than A.

If one adopts MODUS PONENS PERMISSION (even with the modification I just proposed), Complexity fails. In particular, if I believe $\neg\neg A$ and I also believe $\neg\neg A \rightarrow A$, then I can believe A on the basis of believing $\neg\neg A$ and $\neg\neg A \rightarrow A$. Rationality would allow me to believe A (partially) on the basis of believing $\neg\neg A$. One might be tempted to abandon Complexity, anyway.

In addition to these principles involving other connectives, there might be other basing principles involving the logical connectives that appear in the grounding rules (i.e. negation, conjunction, disjunction, quantifiers). Even though it is not easy to find a clear example, this possibility is not ruled out by my proposal. One candidate is UNIVERSAL QUANTIFIER ELIMINATION PERMISSION, i.e. rationality allows you to believe $\varphi(a)$ on the basis of believing $\forall x \varphi(x)$. This is usual in formal sciences, where the generalizations do not come from individual assertions. The following example shows a case where this principle is used:

[16] Broome [2014] p. 191.

[17] Broome [2014] introduces a temporal parameter in the basing permissions. For simplicity, I ignore this aspect of the principles.

Example 4: You believe that 6586867 multiplied by 2 is even on the basis of believing that *every* number multiplied by 2 is even.

Finally, some basing principles might not express permissions, but prohibitions, of the kind "rationality forbids believing A on the basis of believing B". The problem of these new principles, at first sight, is their lack of formality. For example, assume that a basing prohibition states "rationality forbids you to believe A ∧ B on the basis of believing A". This principle, which is intuitively true, has some exceptions: even GROUNDING PERMISSION allows you to believe A ∧ (A ∨ A) on the basis of believing A. It is possible to develop specific and non-formal requirements, such as "rationality forbids you to believe that John is lazy on the basis of believing that John is foreigner and that some foreigners are lazy"[18]. This strategy might be effective, but it is philosophically not very promising (at least from a logical point of view), for it lacks generality.

Complexity

Following the observations of Field [2009], one can add some considerations about complexity to GROUNDING PERMISSION. For, as Broome points out:

> At least one claim seems plausible at first: that it is always permissible to base a belief on other beliefs if the content of the first belief is a logical consequence of the contents of the others. But even this is not so. Suppose the Goldbach Conjecture is a logical consequence of the Peano Axioms – no one knows whether this is so. Even if it is so, rationality does not permit you to believe the Goldbach Conjecture on the basis of believing the Peano Axioms.[19]

This problem applies directly to VALIDITY PERMISSION, but it can also be applied to GROUNDING PERMISSION. It may happen that Γ implies A using grounding rules, but the derivation is too long, so that one is not able to recognize it as valid. Therefore, in these cases you cannot base your belief A on your beliefs Γ: you have no reason to do it.

Therefore, one might add to GROUNDING PERMISSION a consideration regarding the complexity of the inferences. There are many options on the market (as I mentioned in the introduction), but I will adopt a simple restriction:

[18] Ibidem, p. 186.

[19] Ibidem, p. 190.

(**REALISTIC GROUNDING PERMISSION**) If Γ grounds A, and you believe Γ, and *the inference from* Γ *to A is feasible for you*, then rationality permits you to believe A on the basis of believing Γ.[20]

This new principle can respond to Broome's objection about Goldbach conjecture: in cases in which Γ grounds a sentence A, even though you are not able to make this inference, you cannot believe A on the basis of your beliefs Γ. Anyway, I admit that the problem of feasibility for the requirements of logical rationality is much deeper than this; unfortunately, it cannot be addressed in this paper.

Why Grounding?

Admittedly[21], the argument of the paper seems conceptually independent from the deep philosophical concept of *grounding*. As a matter of fact, we just need the grounding rules: these rules could characterize a family of basing permissions. So, why is it necessary to talk about grounding?

Grounding is usually taken to be a *metaphysical* notion, which describes some of the ultimate features of reality. It is supposed to explain the metaphysical priority of some facts over other (more basic) facts: normative facts could be grounded in non-normative facts, semantic facts could be grounded in social and cultural facts, etc.[22] According to Shaffer,[23] grounding "links the worlds across levels"; in other words, it "connects the more fundamental to the less fundamental".

Fine[24] claims that grounding is "central to realist metaphysics", and it could respond to some classic and important questions such as the nature of metaphysical reduction: it can be said (broadly speaking) that a property F can be reduced to a property G whenever the F-facts are grounded on the G-facts. But is there a reason to use such a complex metaphysical notion for logical rational requirements, instead of just relying on the rules?

Well, the first reason for using the notion of grounding is that the rules I proposed are *actually* the rules for grounding (at least in Correia's [2014] and Fine's [2012] formulation). It would be historically and sociologically inaccurate to introduce the same set of rules, without making reference to their original context of formulation. Now, it could still be the case that the notion of logical grounding

[20] This modified basing principle has some similarities with Cherniak's [1981] theory of "minimal rationality", which takes into account the feasibility of the inferences.

[21] This observation was raised by an anonymous referee.

[22] Cf. Correia, Schnieder [2012] p. 1.

[23] Shaffer [2012] p. 122.

[24] Fine [2012] p. 41.

is just sociologically relevant, not conceptually relevant, for this discussion. But this is not the case. Admittedly, metaphysicians claim that grounding is a purely metaphysical notion.[25] However, this does not mean that it cannot play a role in an epistemological principle such as a rational requirement.

Some metaphysical (or at least, not agent-relative) notions are usually taken as relevant for epistemic norms. For example, a norm of *truth* for belief says that one should believe *p* whenever *p* is true. In this case, the norm for belief is external to the agent. A basing permission based on grounding could have the same nature: grounding relations are objective and belong to the real world, but they can introduce (epistemic) basing permissions for particular agents.

Certainly, most authors in the discussion have disregarded a purely epistemic reading of grounding; i.e. grounding cannot be identified with "reasons for belief". As Schnieder claims,[26] looking at a blood stain in a wall is a reason for believing that a crime was committed there; but the stain does not *ground* the fact that there was a crime. However, at this point, it is important to remark that I am not identifying logical grounding with reasons for belief: I am just saying that logical grounding provides some reasons for belief (or strictly speaking, basing permissions). But this is not exhaustive, in two senses: (a) other logical rules may provide basing permissions; (b) other cases of basing permissions could not be motivated by any kind of logical rule. In a nutshell, a connection can be made between the objective notion of grounding, and the epistemic notion of basing permission: (logical) *grounding* provides a family of basing permissions. But this is not to say that grounding is essentially an epistemic notion.

This should not be seen as a miracle. Even though most authors take *grounding* as a primitive metaphysical notion, it is also usually understood as a kind of *explanatory* concept.[27] A grounding explanation responds to the question "why?"[28] but it is not causal; it does not connect an effect to a cause, but it does explain "in virtue of what" something happens. For example, it is correct to say that "2+2=4 or 2+2=5" is true in virtue of the fact that "2+2+4" is true; but the fact that 2+2=4 does not *cause* that 2+2=4 or 2+2=5. Or we can say that a sphere has a power to roll in virtue of its shape; but the spherical shape does not *cause* the things to have the power to roll.[29]

[25] Correia, Schnieder [2012] p. 24.

[26] Schnieder [2011] p. 447.

[27] Fine [2012] p. 37; Audi [2012] p. 101.

[28] Ibidem, p. 104.

[29] Ibidem.

Therefore, there is at least one clear sense in which we can believe the grounding-conclusion on the basis of believing the grounding-premises: the premises explain why the conclusion is true. The possibility of using the very notion of grounding to develop epistemic basing permissions is therefore not particularly surprising. It is an instance of a general truth: that if one explanation is successful, you may rationally believe the *explanandum* on the basis of believing the *explanans*.

4. CONCLUSION

In the first section of this paper, I argued that the usual requirements such as WS VALIDITY or NS VALIDITY cannot capture the permissible ways of believing a contingent sentence on the basis of believing other sentences. The most straightforward ways of making the connection lead to VALIDITY PERMISSION, which is clearly affected by the bootstrapping problem.

I provided an additional logical requirement which is not based on logical consequence but on the notion of *grounding*. GROUNDING PERMISSION is a basing permission, for it explains how we can rationally believe some sentences on the basis of believing other sentences. Given the irreflexivity of grounding, the requirement avoids the bootstrapping objection. Moreover, given the relevance of grounding, it can respond to some paraconsistent arguments. The hyper-intensionality of grounding can capture cases in which *formally* complex sentences are based on formally simpler sentences (even when those sentences have the same content). Finally, I observed that the proposal is compatible with the addition of other logical basing permissions.

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