BETWEEN OCKHAMISM AND THE THIN RED LINE

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Abstract. In this paper we will put forward a novel semantics for future contingents. The idea behind the semantics is to be a compromise position between the 'Ockhamist' semantics, first put forward by Prior [1966]; Thomason [1970] etc., and a version of the Thin Red Line (TRL) semantics recently proposed by Malpass, Wawer [2012]. The new position is able to represent alternative possibilities in two different ways, as actual or counterfactual, which corresponds to a similar distinction in two-dimensional semantics between the primary and secondary intension. We prove a theorem about the notion of validity that results from the new definition, which in the context of the literature about TRL-theories and Ockhamism has some significance.

Keywords: Branching Time, Future Contingents, thin red line, Temporal Logic, Formal Semantics.

1. Introduction

Ever since the pioneering temporal-logical work of Arthur Prior [1957, 1967], much philosophical work has been focused on future contingents, i.e. on the logical treatment of future-tensed statements about events that are not themselves predetermined. The analysis of such terms seems required if we are to avoid being misled to a sort of logical fatalism. This can happen because there are arguments, long known in classical antiquity, which seem to appeal only to plausible logical principles in demonstrating that the future has to be predetermined.[1] In effect, these arguments intend to show that future contingents are logically impossible. One might have their own opinions about whether one thinks that our world is deterministic or not, it seems reasonable to think that indeterministic worlds make sense; such worlds are at least logically possible. Therefore, the task of the philosophical logician here is to show if there can indeed be given a coherent logical and semantic framework in which future contingents can be true without being necessary. These days there are a number of competing options designed to solve this problem, such as 'Ockhamism', supervaluationism and the Thin Red Line, and the overall choice between them largely trades on logical and semantic details

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[1] It is customary to give a nod to Aristotle's *De Interpretatione* 9, and the 'sea-battle' argument, but we are not convinced Aristotle really makes that argument there. See Hintikkaa [1964] for an interesting alternative reading.

of the theories.

What we present here is a new way of conceiving future contingents that attempts to unite two distinct meta-theoretical outlooks about the foundations of branching-time, which we will characterise as 'democracy' and 'monarchy'. The new theory will have the unimaginative name 'New-truth'. New- truth shares certain properties of each of the two meta-theoretical outlooks, a bit like a mixed genetic inheritance that two parents give their offspring. We present it partly because of the novelty of the theory. But in addition it seems noteworthy because, in a particular sense, it immediately improves on one half of its ancestors, and at least as good as the other; in terms of validity, New-truth is just like the democratic alternative (as we will prove later). This means that the choice between New-truth and the democratic theory can only be of 'metaphysics', or some other non-logical idea. The democrats can complain 'we don't like it', but not 'it is logically deficient'.

2. BT and Ockhamist-Semantics

A formal language required to precisely model future contingents would minimally have to include both temporal operators (as they are future), and modal operators (as they are contingent). A simple case would be to start with a standard propositional logic, L (with propositional variables, p_0, p_1, …), and the usual logical connectives (~, &, etc.). We then extend L with the addition of temporal operators (Fp = 'it will be that p', and Pp = 'it was that p'), and a modal operator ($\Diamond p$ = 'it is possible that p'), resulting in the *temporal-modal* language, L_{TM}. With the basic operators, we can define the duals (G, H and □) as follows:

$Gp =_{Def} {\sim}F{\sim}p$

$Hp =_{Def} {\sim}P{\sim}p$

$\Box p =_{Def} {\sim}\Diamond{\sim}p$

A 'future contingent' is a formula of the form Fp, where it is also the case that both $\Diamond Fp$ and $\Diamond{\sim}Fp$. So a future contingent is a prediction, like "It will be that p", made in a certain type of context. That context is one in which it is possible that it will be that p and also possible that it will not be that p. For instance, the prediction "There will be a sea battle tomorrow", made when there are two opposing fleets facing one another across, say, the Aegean Sea, with the admirals deciding whether to attack tomorrow or not, is a future contingent. The debate is about the treatment of the truth-value of such future-tensed propositions in these contexts.

There have been various proposals for how to think of the semantics of such languages. Some authors formalise the semantics of such languages by taking

both the notions of 'worlds' and of 'times' as primitive.[2] Some take worlds (or 'histories') as primitive and define times as sets of histories.[3] The approach used in this paper follows the mainstream crowd[4] in taking times (or 'moments') as primitive and using them to define histories. We see this as a purely technical distinction, with no metaphysical significance.[5] In all these approaches, there are (either as primitive or taking one as primitive and defining the other) both histories and moments, enabling us to represent the modal and temporal operators as involving quantification (over histories and times respectively). The approach favoured here is referred to as 'branching-time' (BT).

A BT-structure is a set of moments, with a particular sort of ordering on them which enables them to be arranged in a 'tree-like' structure.

Definition 1: S - BT structures

S $=_{def}$ (M, <) The two elements of the structure are as follows:

1) M is a non-empty set of moments, {m, m', m''...}
2) < is a binary ordering relation defined on moments, that is partial (asymmetric and transitive) and satisfies the following:

i) *Backward linearity*:
   $\forall$m1, m2, m3: [if (m2 < m1 and m3 < m1), then (m2 < m3 or m3 < m2 or m2 = m3)])

ii) *Connectedness*:
   $\forall$m1, $\forall$m2, $\exists$m3: (m3 ≤ m1 and m3 ≤ m2)]

The restrictions on the ordering ensure that each moment has a linear past, and either a linear or branching future. To make a BT structure (S) into a model (**M**), we add a valuation function.

Definition 2: M – BT models:

M $=_{def}$ (S, V)

V is a function that assigns a subset of M to each $p_i \in L_{TM}$. More complex formulas are recursively defined in the semantics below.

Histories (sometimes called 'branches'), *h*, are maximal linear subsets (ordered with <) of M. Histories therefore represent entire courses of events (or history). *m/h* denotes a pair (*m*, *h*), and in Ockhamism it is tacitly understood that *m* $\in$ *h*, as

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[2] See the 'T x W' theory in Belnap et al. [2001] p. 196–197; or Thomason [2002].

[3] Cf. Zanardo [2004].

[4] Cf. Prior [1967]; Thomason [1970]; Burgess [1979]; Øhrstrøm, Hasle [1995]; Belnap et al. [2001] etc.

[5] Indeed, later in the paper we will argue for a metaphysical priority for histories over moments.

is standard.

While there are various approaches to the semantics of such languages with BT models, we will be using the 'Ockhamist' semantics here as a 'base', from which we will propose a novel parasitic semantics in a minute. Here are the semantic clauses for Ockhamism:

Definition 3: $\vDash_{Ock}$ - Ock-truth

$\mathbf{M}, m/h \vDash_{Ock} p_i$ iff $m \in V(p_i)$

$\mathbf{M}, m/h \vDash_{Ock} \neg\varphi$ iff $\mathbf{M}, m/h \nvDash_{Ock} \varphi$

$\mathbf{M}, m/h \vDash_{Ock} \varphi \wedge \psi$ iff $\mathbf{M}, m/h \vDash_{Ock} \varphi$ and $\mathbf{M}, m/h \vDash_{Ock} \psi$

$\mathbf{M}, m/h \vDash_{Ock} P\varphi$ iff $\exists m' : [m' < m$ and $\mathbf{M}, m'/h \vDash_{Ock} \varphi]$

$\mathbf{M}, m/h \vDash_{Ock} \Diamond\varphi$ iff $\exists h' : [m \in h'$ and $\mathbf{M}, m/h' \vDash_{Ock} \varphi]$

$\mathbf{M}, m/h \vDash_{Ock} F\varphi$ iff $\exists m' : [m' \in h$ and $m < m'$ and $\mathbf{M}, m'/h \vDash_{Ock} \varphi]$

The key aspect of the Ockhamist semantics, a 'point of evaluation' for a formula, is the specification of a model and a moment-history pair (where m ∈ h). Just like with moments, none of the histories has any priority over the rest, which is the 'democratic' tendency in action; each is treated entirely equally.

It should be noted that F is the only clause that makes reference to a moment that is later than the moment in the point of evaluation. This means that it is the only one that can take on different truth-values when different values are given for the *h* parameter. It follows from this that for all formulas, α, that are free of F (or G):

$\mathbf{M}, m/h \vDash_{Ock} \alpha$ iff $\forall h' : \mathbf{M}, m/h' \vDash_{Ock} \alpha$ iff $\mathbf{M}, m/h \vDash_{Ock} \Box \alpha$

The significance of this is that, unless something is to be settled somehow in the future, it is 'now-unpreventable' (as Prior was fond of putting it). If you are now reading this paper, it is now-unpreventable that you are reading it. You could have prevented it at some point in the past, but not any more. Another way of putting this is that the way in which the past is necessary also applies to the present. This logic and semantics captures this insight, and it is characterised by the above equivalences.

The Ockhamist semantics' key novelty is that a history parameter is given for the evaluation of formulas. According to different values of this parameters the same future-tensed statement can take different truth values. The Ockhamist semantics has many attractive logical properties. The system validates such intuitive formulas as 'φ → PFφ', and allows 'indeterminist' formulas such as '◊Fφ ∧ ◊¬Fφ'. As Thomason notes, using both the temporal-modal language L_{TM} and the Ockhamist semantics,

[...] indeterminist frames can be accommodated without sacrificing any orthodox validities. This is good for those who (like me) are not determinists, but feel that these validities are intuitively plausible.[6]

However, despite the attractive logical properties, one may question the use of the history parameter in this semantics. There is a sort of 'flexibility' about which history one chooses to use to evaluate a formula that comes with the 'democratic' nature of the Ockhamist perspective; i.e. no history is distinguished in any way above the rest. And yet, we might think, if the formula is a future contingent, a choice should be made, and that this choice matters. In the context of a genuine future contingent, there will be one history h, say, according to which the future contingent is Ock-true; but there will also be some other history h' which also passes through that very moment, on which the future contingent it is Ock-false. According to the Ockhamist, there is nothing to say which one should be used in order to evaluate the truth-value of the formula. However, one might think that to this extent the semantics does not represent the reality it is trying to model. The values of the history parameter cannot be all equal, because in reality something makes one value distinct from the rest. When we make predictions about future contingents it is not up to us to choose which value of the history parameter to use, and nor is it an arbitrary matter; rather, one way or the other, as history unfolds one possibility gets realised at the expense of the others. This has the consequence that one choice of value for the history parameter turns out to have been right, and the other wrong, even though we could not know this in advance. According to this line of argument, the Ockhamist semantics misrepresents the reality of our real situation by refusing ever to countenance one history over any other; the Ockhamist is too democratic by refusing to privilege any history.[7] Given that the Ockhamist semantics might be vulnerable to this line of attack, let us fish around to see which other attempts have been made at providing the semantics in these situations.

3. Alternatives – TRL, supervaluationism and STRL

An alternative to the Ockhamist semantics is a group of theories which distinguish one course of history over the others, the 'actual course of events', referred to as the 'Thin Red Line' (TRL). If the Ockhamist semantics is democratic, then the TRLs are like monarchies, in the sense that they bestow privilege on just the

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[6] Thomason [2002] p. 215.

[7] See Malpass, Wawer [2012] p. 122–124, for more detail on this problem.

monarch (or the 'actual history'). Typically, TRL-theories build some reference to what is true in the TRL into the truth condition for the future-tense operator.

These theories have historically not come up to the logical standards of Ockhamism. Take, for instance, the TRL-semantics of Braüner, Øhrstrøm, Hasle [1998]. It is an ingenious and sophisticated theory designed to get around various objections proposed by Belnap, Green [1994]. It involves having one TRL at every moment in the indeterministic frame, and a principle of stability governing how those TRLs overlap with one another. However, the complicated formal machinery proves deficient when we consider the formulas it validates. On this semantics, they end up losing the intuitive formula: $H(Gp \rightarrow \Diamond Gp)$.[8] They allow that new possibilities may arise as time passes which were not possible before. The result of this is that one could say truly 'It is impossible that there will be a sea battle in two days', even though the next day it becomes possible, and the day after that it happens to take place. So, one could speak truly when saying it is impossible, even though it happens. Braüner, Øhrstrøm & Hasle attempt to welcome the consequence, but it seems to me to be a tough sacrifice to make.

Another well-known alternative approach is to use the values of the history parameter in the Ockhamist semantics as precisifications in supervaluationism (see Thomason [1970, 2002]). On this account, a formula is super-true at a moment m iff it is Ockhamist-true on all histories passing through that moment:

Definition 4: Super-truth (and super-falsity)

$\mathbf{M}, m \vDash_{Sup} \varphi$ iff $\forall h$: [if $m \in h$, then $\mathbf{M}\ m/h \vDash_{Ock} \varphi$]

$\mathbf{M}, m =\!|_{Sup} \varphi$ iff $\forall h$: [if $m \in h$, then $\mathbf{M}\ m/h \nvDash_{Ock} \varphi$]

This theory has various virtues. For a start, it validates all the same formulas (and no more) as the Ockhamist semantics.[9] It also does away with the history parameter (at least, for super-truth anyway). It is also good if you are inclined to think that future contingents are neither true nor false.[10]

Given that $\mathbf{M}, m \vDash_{Sup} \varphi$ if φ is true on all histories through *m*, this makes the following hold:

$\mathbf{M}, m \vDash_{Sup} \varphi$ iff $\mathbf{M}, m \vDash_{Sup} \Box\varphi$

$\mathbf{M}, m =\!|_{Sup} \varphi$ iff $\mathbf{M}, m \vDash_{Sup} \Box{\sim}\varphi$

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[8] See Braüner, Øhrstrøm, Hasle [1998] p. 7.

[9] See Thomason [1970] p. 274.

[10] It should be noted that for 'super-truth', $\Box\varphi$ (meaning 'φ is true on all histories') is true iff φ is true, and is thus trivial. This also makes $\Diamond$ vacuous. We can consider the Super-Truth language as purged of those operators, as they play no non-trivial role in it.

So super-truth is like necessity and impossibility in the Ockhamist language.

There are doubts about the notion of super-truth that it generates. This version of supervaluationism generates a notion of super-truth that is, in a certain sense, precisely the same as the canonical supervaluationism of Fine [1975], frequently discussed in the vagueness literature. There are many well-known criticisms of this notion, each of which picks up on how much of a departure super-truth is from the classical concept.[11] One example is that of Williamson, who says that for the supervaluationist the notion of semantic consequence (i.e. the preservation of super-truth) rules out such intuitive argument forms as contraposition, conditional proof, argument by cases or reduction ad absurdum.[12] The gains that the semantics has provided us with have had a considerable cost.

Another version of a TRL theory was proposed by Malpass, Wawer in [2012]. They combine a single-TRL view with supervaluationism; they call the view the supervaluational Thin Red Line (STRL). On this view a model, $\mathbf{M}_{TRL}$, is a branching-time model with a distinguished history, the TRL:

Definition 5: TRL-model

> $\mathbf{M}_{TRL} = (S, V, TRL)$

> STRL-truth is defined as follows:

Definition 6: STRL-truth

> $\mathbf{M}_{STRL}$, m $\vDash_{STRL}$ φ iff ($\forall$h: [if m ∈ h, then $\mathbf{M}_{TRL}$ m/h $\vDash_{Ock}$ φ]) or (if m ∈ TRL, then $\mathbf{M}_{TRL}$ m/TRL $\vDash_{Ock}$ φ)
>
> $\mathbf{M}_{STRL}$, m $\dashv_{STRL}$ φ iff ($\forall$h: [if m ∈ h, then $\mathbf{M}_{TRL}$ m/h $\nvDash_{Ock}$ φ]) or (if m ∈ TRL, then $\mathbf{M}_{TRL}$ m/TRL $\nvDash_{Ock}$ φ)

On this view, if a formula is evaluated at a moment on the TRL, then one uses the TRL as the value of the h parameter in the Ockhamist semantics (i.e. STRL-true iff Ock-true); if the moment is off the TRL, then one evaluates according to the supervaluationist procedure. This idea reflects a view about modality, according to which the 'actual' and the 'non-actual' have different properties; the actual enjoys a privileged status, which is familiar to all TRL views. On this view in particular, the consequence is that future contingents evaluated at moments that are not in the TRL are neither STRL-true nor STRL-false. So, keep fixed that I do not actually flip the perfectly fair coin at 5PM, and consider that possible, counterfactual, moment at which I did flip it at 5PM; in this situation there is no answer to

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[11] See Williamson [1994]; Tweedle [2004] or Graff-Fara [2010].

[12] See Williamson [1994] p. 151–152.

which way it lands. If a moment is counterfactual, then at that moment a future contingent has no STRL-truth value. Given that I did not actually flip the coin, this question has no answer. This is the view that STRL embodies.

STRL has a very rigid approach to actuality. STRL can only talk about counterfactual moments as if they were counterfactual; i.e. it is as if STRL-theorists have taken a vow that they will automatically prefix everything they say with the holy mantra: 'The TRL alone is actual...'. To STRL-theorists, they can think about the coin flip that did not actually happen; but they can do so only on the strict condition that it did not actually happen.

A very similar distinction can be found in the foundations of two--dimensional semantics. The basic ideas of 2-D semantics are explained in different terminology depending on which author you are reading, but the basic point is as follows. There is a sense in which "2 + 2 = 5" is impossible. Given the meanings that the terms have in the actual world, there is no world in which it is true. However, there is another sense in which there is a world at which "2 + 2 = 5" is true. In some world, its inhabitants use the word "5" to mean what we mean by "4", and thus at this world (with the inhabitants' meanings) 2 + 2 does equal 5. The lesson is this: if we keep fixed the meanings of the terms as we find them (or 'fix' them) in the actual world, then "2 + 2 = 5" is impossible, but if we allow the meanings to change when we consider different worlds, then it becomes contingent. Allowing the meanings to change as we consider the other worlds (the 'primary intension') has been likened by David Chalmers to 'considering those worlds as if they were actual'. Keeping the references fixed to what they are in the actual world when we consider the other worlds (the 'secondary intension') is to consider them as 'counterfactuals'. Chalmers makes the following comments about secondary intensions, but these comments could easily be describing how to think of m according to STRL if m ∉ TRL:

> When we consider these scenarios, we are not considering them as epistemic possibilities: as ways things might be. Rather, we are acknowledging that the character of the actual world is fixed, and are considering these possibilities in the subjunctive mood: as ways things might have been. That is, rather than considering the possibilities as actual (as with epistemic possibilities), we are considering them as counterfactual.[13]

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[13] Chalmers [2002].

On this understanding of the primary vs. secondary distinction, it seems that one way to describe how STRL is 'rigid' with actuality would be to say that it is stuck with secondary intension. It can never consider the 'possibilities as actual', but has to insist that "the character of the actual world is fixed". It is this rigidity which we believe is behind Belnap et al.'s worries that TRL theory has "troubles with actuality".[14]

This proposal, although it could be viewed as a version of supervaluationism, is not vulnerable to the Williamson/Tweedle/Graff-Fara objections, as is shown in Malpass, Wawer [2012] and Malpass [2013]. On this view, everything that is Ockhamist-valid is also STRL-valid. This means that one gets 'indeterminist frames' without sacrificing any of the 'orthodox validities' that the Ockhamist semantics enjoys. In particular, the key to avoiding the Williamson/Tweedle/Graff-Fara objections was that there are moments at which both "Fp" and "◊¬Fp" are STRL-true. This is the definition of a genuine future contingent, and so at this point the future is 'open' with respect to p. Therefore, addition of the TRL and the STRL semantics does not straightforwardly lead to fatalism or determinism.[15]

While everything that is Ock-valid is STRL valid, there are, however, formulas that are STRL-valid that are not Ockhamist-valid. The formula 'Fφ → FFφ', for instance, is only Ockhamist-valid in dense frames; but for STRL it is also valid in frames that are 'pseudo-dense' (i.e. dense everywhere apart from immediately between the TRL and the rest of the structure). It is unclear at present the extent to which the STRL notion of validity departs from the Ockhamist one; STRL 'contains' Ock-truth, but that is all that has been said conclusively at the moment.[16]

All this means that the logical properties of contemporary TRL views either exclude some validity you do want (like Braüner, Øhrstrøm, Halse), or bring in a very rigid notion of actuality and give you some validity you do not want (like Malpass and Wawer), when compared to the Ockhamist alternative.

Here, we will propose a new semantics which mimics the STRL distribution of truth-values for formulas (i.e. upholds a distinction between the actual and the non-actual), but crucially gives up the notion of a fixed TRL altogether, and therefore does not suffer from the rigid notion of actuality. Therefore it is a compromise position; it has the ('monarchic') notion of 'privileging' that TRL theorists like, and

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[14] See in particular Belnap et al. [2001] p. 163.

[15] Perhaps one way of explaining why it is that so many people seem to think that TRL indicates determinism or fatalism of some sort is that they are worrying about the closely related issue that most TRL-theories, like STRL, are stuck with secondary intension only. Strictly speaking, though, that issue is separate to the issue of whether the future is 'open'.

[16] See Malpass, Wawer [2012] p. 159, appendix (2a), for further discussion.

the ('democratic') notion of 'mobility' that the Ockhamists like. We will prove that the notion of validity on this new semantics is exactly that of the Ockhamist semantics. Therefore, this position should be attractive to both sides of the debate over the TRL, as it has principles that each party wants to endorse, and it does this without sacrificing any logical validities.

4. New-truth

In what follows, we will present a novel way of thinking about the semantics of future contingents, which we shall call 'New-truth'. The approach outlined will endorse a split between Ock-truth and New-truth, similar to the split between Ock-truth and Sup-truth; the New-truth truth-condition will involve reference to what is true at the Ock-level. This is the same in the case of supervaluationism and STRL. However, the point of evaluation in the New-truth semantics will still have a history parameter, unlike in supervaluationism or STRL, and so the cases are properly distinct.

In New-truth formulas still require both a moment and a history, but the main distinctive novelty is that we relax the requirement that $m \in h$. This provides the semantics with a richer framework to evaluate formulas in, as we shall see in a moment.

A 'New-truth' point of evaluation would be: $\mathbf{M}, m, h \vDash_{New} \varphi$. Remember, it might be that $m \notin h$. Now we can define 'New-truth', and it will be somewhat like Malpass and Wawer's STRL-truth, but using the mobile h-value in the New-truth point of evaluation in place of their 'special' TRL-value. So New-truth 'privileges' a history, exactly like STRL did in a fixed way; but unlike STRL, which one it privileges depends on where you start the evaluation, and so which history is privileged is 'mobile' (we will explain and discuss this more in a moment). The definition of 'New-truth' goes as follows:

Definition 7: New-truth

$\mathbf{M}, m, h \vDash_{New} \varphi$ iff

$$(m \notin h \text{ and } \forall h': [\text{if } m \in h', \text{ then } \mathbf{M}, m/h' \vDash_{Ock} \varphi])$$
$$\text{or } (m \in h \text{ and } \mathbf{M}, m/h \vDash_{Ock} \varphi)$$

$\mathbf{M}, m, h \dashv_{New} \varphi$ iff

$$(m \notin h \text{ and } \forall h': [\text{if } m \in h', \text{ then } \mathbf{M}, m/h' \nvDash_{Ock} \varphi])$$
$$\text{or } (m \in h \text{ and } \mathbf{M}, m/h \nvDash_{Ock} \varphi)$$

(At $\mathbf{M}, m, h$ it is New-true that φ iff either (m is not in h, but for all histories, h', which do pass through m, φ is Ock-true at $\mathbf{M}, m/h'$), or (m is in h and φ is Ock-true at $\mathbf{M}, m/h$))

(At **M**, m, h it is New-false that φ iff either (m is not in h, but for all histories, h', which do pass through m, φ is not Ock-true at **M**, m/h'), or (m is in h and φ is not Ock-true at **M**, m/h))

If m ∈ h, then either (**M**, m, h ⊨_New φ) or (**M** m, h ⊣_New φ); i.e. if m ∈ h, then all formulas are either New-true or New-false. On the other hand, if m ∉ h, then there are exceptions; i.e. formulas that are neither New-true nor New-false. The only formulas that come out neither New-true nor New-false are future contingents at moments such that m ∉ h. Fixing a value for the h parameter means that we are imposing a condition like 'given that h is actual...'.

According to this view: given that h is actual, if m ∈ h, then m is 'actual'. On the other hand, given that h is actual, if m ∉ h, then m is 'counterfactual'. Being able to represent both sides of this distinction is the main difference between the New-semantics on the one hand, and the Ockhamist semantics and STRL on the other. On the New-semantics a moment can be considered as either actual or counterfactual.

In STRL there is a very similar distinction between those moments that are in the TRL and those that are not in the TRL. However, in STRL the TRL is fixed as part of the structure itself. Moments in the TRL are actual, and moments not in TRL are counterfactual. However, STRL cannot change which history it considers as actual, which is the basis of the "troubles with actuality" we saw above. So STRL has the 2-D-like distinction between actual and counterfactual (which is good), but has a fixed idea of what is actual (which is bad).

In Ockhamism, on the other hand, assigning a value to the history parameter is to provisionally consider that history as actual; thus Ockhamism is flexible with respect to which history is actual (and thus suffers from no "troubles"). However, each moment is only ever called into use when it is paired up with a history that passes through it, and so each moment is only ever considered actual with respect to its partner history. So Ockhamism has a flexible idea of what is actual (which is good), but has no 2-D-like distinction between the actual and the counterfactual (which is bad).

In New-truth, we get the benefit of the distinction between the actual and counterfactual, but we get it with the benefit of the mobility of the history parameter that we get in Ockhamism. On this theory, then we get the best of both worlds. In New-truth, each history thinks that it is a Thin Red Line in one of Malpass and Wawer's STRL models. To further the metaphor used so far in this paper, it is, as it were, a democracy where every citizen thinks they alone are the monarch. Therefore, the New-semantics has properties that should be attractive to both sides of the debate; it is halfway between Ockhamism and the Thin Red Line.

5. Actuality

One very simple application of the New-truth semantics is to the definition of an 'actuality' operator, usually symbolised "@". We will show how a very simple idea of how to define @ in the Ockhamist semantics leads straightforwardly to an operator that behaves on the New-truth level just like an operator defined by Belnap et al.,[17] called "Actually$_2$", which we shall symbolise as "@$_2$". On the Belnap-approach, there is another moment parameter, the 'moment of use', "m_c", which allows a 2-dimensional evaluation of formulas, such as those involving actuality operators. We claim that on New-truth one gets the same behaviour for the @ operator, but without the extra parameter (because the framework is already explicitly 2-D). Belnap et al.'s definition of @$_2$ (simplified slightly), is as follows:

Definition 8: @$_2$

$\mathbf{M}, m_c, m/h \vDash$ @$_2$ φ iff

($m_c \in h$ and $\mathbf{M}, m_c, m_c /h \vDash \varphi$) or $\forall h'$: [if $m_c \in h'$, then $\mathbf{M}, m_c, m_c /h' \vDash \varphi$])

Obviously, we have not provided all the details with which to properly grasp this definition. However, they informally explain how to think of @$_2$ as follows:

> On this variant, one considers whether or not the moment of use sits on the current history. If so, the variant ties actuality to what is plain true at the moment of use on the current history. If, however, the moment of use does not sit on the current history, the variant ties actuality to what is settled true at the moment of use.[18]

It is clear that Belnap et al's definition has something close to New-truth about it. For New-truth is already essentially "what is plain true" (i.e. Ock-true) so long as $m \in h$, and what is "settled true" (i.e. true on all values of h) if $m \notin h$.

Because New-truth already has the patterns of behaviour that Belnap's @$_2$ operator has, we can get an actuality operator to behave like this very easily on the New-truth picture. All one has to do is define @ on the Ockhamist level as follows:

Definition 9: @

$\mathbf{M}, m/h \vDash_{Ock}$ @φ iff $\mathbf{M}, m/h \vDash_{Ock} \varphi$

At the Ockhamist level, the @ operator is thus entirely trivial. On the New-truth level, if m is 'actual' with respect to h, then the formulas that @ can prefix

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[17] Belnap et al. [2001] p. 246.

[18] Ibidem.

are those formulas that are Ock-true, and is still trivial. However, if m is 'counter-factual' with respect to h (i.e. if m ∉ h), then those formulas that can be prefixed with an @ are only those that are Ock-true on every history through m. This is precisely what Belnap et al.'s operator does.

It is interesting to see how such a simple definition of @ allows us to make an operator that fits naturally into the picture and has properties very similar to Belnap et al.'s independently constructed formalisation. We claim that New-semantics is such a rich framework that it might naturally prove itself to have many other interesting applications in the long run, perhaps in the evaluation of two-place counterfactual connectives. We include this section mainly to demonstrate with a particularly easy example that it has these other potential applications.

6. Validity

The much discussed debate between the TRL-theorists (such as Peter Øhr-strøm) and the Ockhamists (such as Nuel Belnap) took the form of trying to maintain various logical validities independently deemed to be plausible and intuitively true. The general consensus is that the TRL theories have not come up to the standards of the Ockhamists in this arena. We will now prove that New-truth is precisely as good as the Ockhamist alternative in this respect.

The notion of validity used here will be a standard one, found for instance in Malpass, Wawer [2012]:

Definition 10: Ock-Validity:

A formula φ is Ock-valid in a BT-structure, S = (M, <), (i.e. S $\vDash_{Ock}$ φ) iff for every model, **M** = (M, <, V), based-on S and for every moment/history pair (i.e. with m ∈ h) m/h ∈ S, **M** m/h $\vDash_{Ock}$ φ.

(A formula is Ock-valid iff it is Ock-true at every Ockhamist point of evaluation)

Definition 11: New-validity:

A formula φ is New-valid in a BT-structure S = (M, <), (i.e. S $\vDash_{New}$ φ) iff for every model, **M** = (M, <, V), based-on S and for every pair (m, h) ∈ S, **M** m, h $\vDash_{New}$ φ.

(A formula is New-valid iff it is New-true at every New-truth point of evaluation)

Given these standard notions of validity (i.e. truth at all points of evaluation), we can express the claim that New-truth is logically equivalent to Ock-truth.

Theorem 1:

For all BT-structures, S:

S ⊨$_{New}$ φ iff S ⊨$_{Ock}$ φ

Proof:

Right to left: S ⊨$_{Ock}$ φ ⇒ S ⊨$_{New}$ φ.

Argue contrapositively and assume S ⊭$_{New}$ φ. Thus, by the definition of New-validity (Def. 10), there is some model **M** based on S, and some (m, h) pair ∈ S, such that **M** m, h ⊭$_{New}$ φ. If m ∈ h then it follows from the second disjunct of the definition of New-truth (Def. 7) that **M** m/h ⊭$_{Ock}$ φ. If m ∉ h then it follows from the first disjunct that there is an h' such that m ∈ h' and **M** m/h' ⊭$_{Ock}$ φ. On either case there is an Ockhamist point of evaluation according to which φ is not Ock-true. Thus, according to the definition of Ock-validity (Def. 11), it follows that S ⊭$_{Ock}$ φ.

Left to right: S ⊨$_{New}$ φ ⇒ S ⊨$_{Ock}$ φ

Again, argue contrapositively and assume S ⊭$_{Ock}$ φ. Then, by the definition of Ock-validity (Def. 10), there is some model **M** based on S, and some m/h pair ∈ S, such that M m/h ⊭$_{Ock}$ φ. Thus, by the second disjunct of the definition of New-truth (Def. 7), **M** m, h ⊭$_{New}$ φ. Thus, there is a New-truth point of evaluation according to which φ is not New-true. Thus, according to the definition of New-validity (Def. 11), S ⊭$_{New}$ φ. □

6. Conclusion

The main move in this paper was to consider what happens if one relaxes the requirement that a moment is part of its partner history in a point of evaluation for a formula. What we have shown is that we can devise semantics that is equivalent to the Ockhamist one in terms of validity. It might be wondered whether this was a purely formal exercise, or whether there is any other motivation for such an endeavor. We think that there is an interesting idea which underlies this semantic theory, and that has to do with the deeper metaphysical picture implied by it. Prior, who developed temporal logic, was a presentist, and this favouring of the present moment has been retained in the metaphysical picture behind the Ockhamist semantics. On that theory, we claim, the moment in the moment/history pair is the metaphysically prior entity. While there is a sort of relativity when it comes to which history to use as the value of the history parameter, the choice of moment to use for the moment parameter is determined by the context. If I say 'There will be a sea battle tomorrow', then when I am saying it, it is part of the context, but, on Ockhamism, the value of the history parameter is not

given by the context. This means that there is a sort of 'metaphysics of context' implicit in the Ockhamist semantics, and it involves, if not outright presentism, then at least priority of the moment over the history. On New-truth, in contrast, the priority is reversed, and the history is the metaphysically prior entity, at least on the intended interpretation given here. The way we think of it involves determining whether the moment is thought of as actual or non-actual depending on whether it is part of the history of evaluation or not. The intended reading is: given that h is actual, m is actual if m $\in$ h, and m is non-actual if m $\notin$ h. This metaphysics is much closer to the TRL metaphysics, which also seems to metaphysically privilege histories over moments.

One can stand back from the formal technicalities involved in this debate and see the Ockhamist and supervaluationist theories as both expressions of presentism (or the 'A-theory'), and TRL, STRL and New-truth theories as expressions of eternalism (or the 'B-theory'). Thus, part of the motivation for this theory is to find a technically respectable eternalist theory of future contingents, and we believe that the New-truth validity theorem given here goes a long way towards establishing this result.

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