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DUMMETT'S FORWARD ROAD TO FREGE AND TO INTUITIONISM

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First, I will briefly summarize the earlier discussion. Although Dummett does sometimes say that for Frege there is only a forward road from sense to reference, I had argued at length that there are texts in Dummett which imply or appear to imply Dummett holds that for Frege there is a backward road from our being able to identify references to our being able to identify senses. Dummett says in his reply to me that he has always known that for Frege there is only a forward road from sense to reference (Dummett [2007] p. 114). Thus Dummett makes it clear we agree that there is only a forward road from senses to references in the *cognitive* order in Frege, in keeping with Russell's famous "no backward road" thesis about Frege in particular and about connotations and denotations in general (Russell [1971b] p. 50). What is new for Dummett is that he now also agrees with me that for Frege no senses can be objects or functions. He recants his view that senses are objects, and appears to restate my scholarly and conceptual arguments against that view in his own way (Dummett [2007] p. 122). On the scholarly side, Dummett appears to agree with me now that where Frege seems to say that all entities are either objects or functions, the texts are, taken in context, only about customary references (*ibidem*). Since Dummett holds (correctly in my view) that senses are entities, but are not functions, these texts had mistakenly led him to infer that senses must be objects. Dummett also appears to agree with me now that where Frege seems to say that singular definite descriptions always refer to objects, the text is, taken in context, only about customary references (see *ibidem*). Since Dummett holds (correctly in my view) that "the sense of expression 'A'" refers to a sense, this text had mistakenly led him to infer (again) that senses must be objects. It seems, then, that Dummett and I now agree that "the sense of expression 'A'" belongs to indirect speech about senses, not to customary speech about customary references. Dummett also offers a new argument which I did not. The gist of it is that since functions cannot be senses, and since functions and objects go hand in hand, objects cannot be senses either (*ibidem*). I accept this new argument of his. Finally, Dummett's conceptual argument that it is misleading to regard senses as objects forming a "halfway station" on the way *to* objects (*ibidem*)

is a figurative version of my more literal and general argument that senses cannot function as customary references and remain senses, since senses as such are presenters of other things, and customary references as such are not (Dejnožka [2007] p. 77)[1]. But there is one point on which we still disagree. While we agree that no senses can be objects for Frege, I hold that this is also Frege's expressly stated view. But Dummett is not fully convinced that Frege actually holds, believes, or is even necessarily aware that that no senses can be objects (Dummett [2007] p. 123--124). I will simply say here that I believe Frege states or at least clearly implies this view when he says in "On Sense and Reference," "A truth-value cannot be a part of a thought any more than, say, the Sun can, for it is not a sense but an object" (Frege [1971] p. 64; see Dejnožka [2007] p. 89-93). This concludes my summary of our earlier discussion.

As Russell says, "there is no backward road" from references to senses (Russell [1971b] p. 5). That is because any one reference can be presented via indefinitely many senses. How did I come to think Dummett said there is a backward road from customary references to senses in the cognitive order in Frege? Dummett said "the systematic theory of *Bedeutung* provided a basis for explaining in what the *Sinn* of an expression should be taken to consist, namely the way in which its *Bedeutung* is given to a competent speaker of the language" (Dummett [1995] p. 4-5). He said, "Frege explained the *Sinn* of an expression of his symbolic language as determined by the stipulations specifying its *Bedeutung*" (*ibidem*, p. 14). I had thought he was saying that Frege derives senses from customary references, that reference determines sense. I think now he was saying that Frege determines or arrives at the sense not from the reference itself, but instead from the *way* the reference is *determined*. This is better, since it does not take the backward road. But I think even this is strictly wrong. I think the thought simply *is* the way the truth-condition is cognitively determined, and as Dummett himself sometimes says, the sense simply *is* the way the reference is cognitively determined. More precisely, the way the reference is determined is the mode of presentation the sense contains. Thus there is no forward *or* backward road from the way the reference is determined to the sense, or more precisely, to the mode of presentation it contains, since they are the same. This is perfectly consistent with there being only a forward road from to sense to reference in the cognitive order, since identifying the way the reference is determined with the mode of presentation is only on the side of sense.

[1] We agree, of course, that n + 1-level senses can present n-level senses in indirect speech.

Dummett (*ibidem*, p. 19) says that since I admit only a backward road from reference to sense in the *ontological* order in *Grundgesetze* I § 32, meaning that for Frege the notion of a way of presenting a thing is logically dependent on the notion of a thing, my view fails to explain how specific senses are identified. He is right that the backward road that I find there fails to explain that. It was not intended to. I also admit a forward road from sense to reference in the *cognitive* order in § 32, and indeed in all of Frege's sense-reference distinction writings, meaning that for Frege the identification of specific senses is logically prior to the identification of specific references. That was the main point of my paper. Indeed, where Dummett says we "determine" or "arrive at" the sense from the way the reference is determined, I say the way a reference is determined *is* the sense, or more precisely, *is* the mode of presentation the sense contains. That is how I explain how specific senses are identified; and my explanation is direct and immediate, not involving Dummett's process of determination or arrival.

I am puzzled why Dummett thinks we "arrive at" or "determine" thoughts from the way truth-conditions are determined, when the case is really one of identity. This is what originally confused me, and he still speaks this way in his reply (Dummett [2007] p. 119, saying "arrive at" twice). Of course, one can trivially "arrive at" or "determine" or "derive" P from P itself, but here it would be far clearer to say that P simply *is* P, and that there is no arriving or determining or deriving at all. For all thoughts are senses, and their truth-conditions are logically composed of the references of their constituent subject- and predicate-senses. And a sense, or more precisely the mode of presentation it contains, simply *is* the way a reference is presented. Frege expressly says the sense contains the mode of presentation. And even if Frege had not said that, it would seem more in keeping with the forward road to go from the sense to the way the reference is determined and then to the reference, than to go in the opposite direction. Yet Dummett's interpretation still seems to be going in the backward direction here, if not from references themselves, then from the way references are determined to senses. But it is not a backward road from reference to sense, since it does not start from references themselves. Also, sense is the obscure notion that needs explaining. Frege takes a sense to be an abstract, timeless connotative meaning expressed by a name, which can either be subjectival ("complete") or predicative ("incomplete") depending on whether the name is a logical subject or a logical predicate, and whose primary constituent is a mode of presentation, meaning a way of identifying the reference of the name, if the name has a reference. A sense is essentially linguistic due to its being expressible by a name.

There is a process of arriving at or "working out" (*ibidem*) complex senses from simple ones, and there is a converse analytical process of arriving at simple senses from complex ones. But there is no process of "arriving at" (modes of presentation contained by) senses from ways of determining references. That is simply what they are.

I am not saying that "sense" and "way a reference is determined" are *synonymous* for Frege. Quite the opposite. In Fregean terms, it would be better to speak of explanation than of definition of a sense as a way a reference is determined. For "sense" would seem indefinable for Frege, since it is one of his basic notions. For Frege, to explain is to state what something is, but in a way that falls short of definition. Explanations are really informative identifications, and as such always involve two different senses. "Way a reference is determined" is not synonymous with "mode of presentation." That is because "way a reference is determined" is essentially linguistic due to its determining a reference in language, while on its face "mode of presentation" is essentially cognitive or phenomenological. This is especially clear if we grasp modes of presentation nonlinguistically in sense-perception, as when we see the same physical object from different angles or under different conditions[2].

I retract my agreement with Dummett's view that "the *Sinn* of an expression should be taken to consist [in] the way in which its *Bedeutung* is given to a competent speaker of the language" (Dummett [1995] p. 4-5)[3]. There are two reasons for this. First, whenever Frege introduces his notion of sense, speaker competence is never mentioned or implied. Second, Frege says that speakers can communicate even if they understand the sense of a name differently (Frege [1971] p. 58 n.*). That is, two competent speakers can communicate using different senses to refer to the same reference. And their equally competent use of the same name can scarcely characterize or even help identify which sense they express, since the-

[2] Dummett finds no significant difference between senses and modes of presentation. I find that our grasp of modes of presentation is often merely perceptual. Senses are always garbed in language, and we do not always speak about the objects we perceive. No doubt perceptual modes of presentation and perception itself, as opposed to mere sensation, are in general logically posterior to language for Frege. But we do not *have* specific names in our language for all the objects presented to us in perception. I also think that one and the same mode of presentation can be contained in either a complete sense or an incomplete sense, depending on how we split a sentence into logical subject and logical predicate. In any case, Frege expressly distinguishes a sense from the mode of presentation it contains, and I am concerned to examine why he goes out of his way to do so, while Dummett finds nothing of interest in the distinction. I call it Frege's assimilation of perceptual modes of presentation to (or more precisely within) linguistic senses. The excitement begins when we move to abstract modes of presentation for abstract objects.

[3] My agreement was limited to explaining what senses are in terms of how they function as presenters of things, i.e., to Frege's backward road in the *ontological* order (of dependence).

re *is* no one sense they both express. That Frege says we ought to avoid such everyday ambiguities of sense in the formal notation does not detract from my point. Frege does not mention speaker competence even when he explains sense in *Grundgesetze* I § 32. The reason should be clear. He is concerned with stipulations of sense in the formal notation, and these stipulations are a logical precondition of competent use of the notation. There can be no competent use of the notation if no senses are stipulated for its names in the first place. And "working out" complex senses presupposes some already given simple senses. Of course, there is such a thing as competence in explicating or defining senses. But that is logically prior to speaker competence in the discursive sense Dummett has in mind. Speaker competence is logically posterior to the introduction of senses even in ordinary language. Speaker competence can only be derivatively defined as use in accordance with already introduced, i.e. already identified, senses. This would be so even if senses could only be introduced within statements. But for Frege, definitions (stipulations) and prelinguistic explications are not statements. They are not truths.

Dummett and I agree that the key texts, *Grundgesetze* I §§ 29–32, concern stipulation (*Festsetzung*). I tentatively agree with Dummett on where explication (*Erläuterung*) probably occurs in *Grundgesetze*. In my [2007], I offered no opinion on whether Frege explicates anything in that work, or whether the explications were in the metalanguage (Frege's semi-formal German, or even ordinary German), since as I said, it did not matter to my issues. Dummett is right that indefinable notions must be explicated (Dummett [2007] p. 116). But undefined notions must be explicated too, since they are not defined. Frege's explanations (*Erklärungen*) introduce nothing, since they are given of terms that already have sense, e.g., Frege's explanation of identity as indiscernibility. A stipulation is a stipulative definition. All stipulations are definitions, hence all stipulated terms are defined terms. Dummett says that stipulations are of undefined terms (*ibidem*). Perhaps he means they are undefined until we stipulate what they are to mean. Or perhaps he does not think, or does not think Frege thinks, that stipulations are definitions. But Frege says all his definitions are stipulations.

Regarded as a whole, a thought is a way of presenting a truth-value. But regarded as split up into parts, a thought is a way of presenting the correspondingly split up specific truth-condition it cognitively determines. In *Grundgesetze*, when Frege stipulates the (specific sort of) truth-condition for a (form of) formal sentence in which a certain primitive expression occurs, he conveys to us a thought in his metalanguage, semi-formal German, and the thought presents the truth-condition. He does not *state* that the formal sentence is to express this thought, and

is thereby to have this truth-condition. Rather, he states what the *truth-condition* is to be, and this metalinguistic statement conveys (not: expresses) the thought which presents the truth-condition, and which is also to be the thought expressed by the formal sentence. That is, he *uses* the thought to do this; he does not *mention* the thought. On his own sense-reference distinction, what else could he be doing? Thus unlike Dummett, I never thought "we might expect that Frege would first [expressly] stipulate or explain the senses of the primitive expressions" (Dummett [2007] p. 114). For Frege does that by use, not by mention.

Dummett says I hold that all truth-conditions are extensional. This is true only in my sense (1) of "extensional," meaning truth-functional. It is not at all true in my sense (2) of "extensional," meaning not being a way of presenting something. It is precisely Frege's systematic shift of reference to senses in what would otherwise be referentially opaque contexts that makes even sentences involving such contexts truth-functional, even though their truth-conditions include senses as logical parts. My statement, "For a truth-condition consists of extensional in sense (2) references," was in the context of a five-paragraph discussion of *customary* sense and *customary* reference. I did not expressly say "a customary truth-condition" at this point, but you can see I meant that from the end of the paragraph, where I say that such references are what "make a *customary* statement true or false" [my 2007, p. 62, my new emphasis]. A few pages later, I expressly allow truth-conditions to have senses as logical parts. I say, "If we enrich L so as to include sentences expressing propositional attitudes, then thoughts would be logical parts of the truth-conditions of L" (Dejnožka [2007] p. 70). I agree with Dummett that truth-conditions are not entities for Frege. If they were, they would be facts more or less in Russell's sense, and Frege rejects such facts (see *ibidem*, p. 70, 105 n. 11). But Dummett misunderstands my distinction between general and specific truth-conditions. He says, "All true [analytic] sentences...would therefore have the same truth-condition, on Dejnožka's understanding of 'truth-condition'" (Dummett [2007] p. 121). This is not true. I said:

> But even if necessary truth is the same as purely general truth for Frege, the *specific* or *proper* truth-conditions of "2 + 2 = 4" and "$(x)(x = x)$" are different, consisting of different specific functions and objects in different specific relationships. Strictly, even "2 + 2 = 4" and "4 – 2 = 2" have different truth-conditions and express different thoughts (Dejnožka [2007] p. 69).

Thus I hold that even necessarily true non-synonymous sentences express different specific thoughts which determine different specific truth-conditions. To say they have the same *general* truth-condition, true under any condition, is merely to

say they are all true *under all possible empirical circumstances*. I apologize for any confusion, but surely that is a familiar way of putting it, and surely Frege holds that view. But Dummett thinks I infer from the extensionality (in which of the four classic senses of "extensional" I described he does not say, but I imagine it is the truth-functional sense) of truth-conditions that all analytic truths have the same truth-condition!

Frege's view on necessary truth in *Begriffsschrift* § 4 does not distinguish between general truth as such and general truth that is true in virtue of its logical form. This suggests he thinks that general truth *is* true in virtue of its logical form, namely, in virtue of its merely being general in form. Russell offers a counterexample: "There exist at least 30,000 objects" is a purely general but contingent truth (Russell [1971c] p. 240). Of course, Frege surely would find the existence of infinitely many natural numbers logically necessary, insofar as they are abstract objects for him. But as a point of scholarship, he still fails to distinguish merely general truth from logically necessary truth. At least, I see nothing in § 4, or in the corresponding *Grundlagen* passage, to show that he *means* by "necessary" or "analytic" truth anything more than purely general truth, or truth derivable from such truth. I suspect that Frege is naively thinking, as Russell did before discovering his counterexample, that surely "logically necessary truth" is best analyzed as 'truth purely general in logical form'. What else could it be, and how else could necessary truth be known? The slide from "invariant" to "necessary" is indeed invidious. But Frege also says in § 4 that it is possible that this crow is white, if not all crows are nonwhite. This suggests an invariance theory of necessity even for what most of us would consider contingent truths. *Grundgesetze* II § 126 suggests why he might think so. There he basically indicates that for him there is no such thing as a merely possible object. This suggests in turn that for him the only possible objects are actual objects, so that if all actual crows are black, then all possible crows are black[4].

I agree that we can *call* expressions of the form "$a = b$" 'identity statements', expressions of the form "a" and "b" 'object-names', and the expression "î = ć" 'name of the identity relation'. But the anti-formalist Frege would be the first to tell us that *mere labels* such as "identity statement," "object-name," and "name of the identity relation" fail to express their normally intended sense if they are descriptions of syntactical form alone. Thus my criticism of Dummett remains, *pace*

[4] I am not alone in seeing Frege as equating necessity with generality in *Begriffsschrift* § 4. See Haaparanta ([1985] p. 38-40, 46 n. 26).

Dummett ([2007] p. 116-117). In fact, Dummett is still doing the same thing I object to. He says in his reply:

> But the form of a sentence is a syntactical feature. Frege gives a very clear exposition of the syntactical formation of sentences of his formal language. From it we are enabled to pick out proper names (singular terms), expressions for first-level functions of one argument, [etc.] We can, for example, refer to sentences of the form "*s* = *t*," where *s* and *t* are singular terms. In doing so, we are not assuming it as known that "=" is a sign of identity, or that we even yet have the concept of identity (*ibidem*, p. 117).

It is commonplace for logicians to say things like, "Let signs of the form 'a', 'b', 'c',... be proper names of objects." But calling something a proper name does not make it one. Proper names do not live by syntax, i.e., by physical shape, alone. Frege says a proper name such as "Nausicaä" must *behave* as a proper name. He means that there are semantic requirements. He requires that proper names express complete senses. He also requires, in a scientific notation that is used to assert truths, that proper names have references. He even requires proof that his proper names denote something before he admits them as proper names [*Grundgesetze* I § 29]. Calling *s* and *t* proper names does not make them proper names any more than calling "=" the identity sign makes it a name of the identity relation. As Wittgenstein would say, baptism is but the first step on the road to namehood. And if we do not yet have the concept of identity, it is no use trying to fix the sense of an identity for our proper names, nor to determine the reference of proper names, that is, whether two proper names refer to the same object, that is, whether "*s* = *t*" is true. Here Dummett is trying to pull proper names out of the hat of syntax. Of course, Dummett discusses semantics *too*. But there is no road from syntax to semantics, no road from physical shape to proper namehood, and no derivation of the identity sign or identity statements from mere marks on paper as such. That is because any one physical sign can be used in indefinitely many ways.

Dummett is puzzled by my view on saturation. He says that if senses are not objects or functions, then they should not be saturated or unsaturated in the way objects and functions are. He may be right that mere Fregean thoughts need no ontological "glue" to hold them together. But this is Frege criticism, not Frege scholarship. I quoted Frege as indicating that for him, saturation is just as basic to sense as it is to reference, and even that "The words 'unsaturated' and 'predicative' seem *more* suited to the sense than the meaning [i.e. reference]" (Frege [1979] p. 119 n.*, my old emphasis). Dummett finds nothing wrong with these texts. If Dummett has found a problem here, it is Frege's, not mine. Of course, "predica-

tive" seems obviously more suited to *senses* expressed by linguistic predicates than to predicates' *references*, which are properties in the world. (Frege equates concepts with properties.) I imagine that "unsaturated" seems more suited to senses because all senses are timeless and unalterable, so that the unsaturated nature of unsaturated senses is a timeless and unalterable ontological "glue" suited to bind timeless and unalterable entities. In contrast, even though all functions and all *abstract* objects are timeless and unalterable, *concrete* objects exist in time and are subject to change. Thus the unsaturatedness of functions as such must allow of alteration. For example, the unsaturatedness of green must bind green to a green apple, but not so strongly that the apple cannot become red. But I think Dummett is really confusing two views of mine. I hold just as much as he does that "The incompleteness [or unsaturatedness] of a predicate [consists in] its being the sense of a predicate" (Dummett [2007] p. 122), and that therefore "the incompleteness of the sense of a predicate or functional expression is of a different nature than that of a concept or function" (*ibidem*, p. 123). But I take that view to concern *only* the fact that a sense's incompleteness (or completeness) is what makes the sense *linguistic*. (I hold that the only other logical ingredient of a sense is the mode of presentation it contains, and that its mode of presentation is, as such, cognitive as opposed to linguistic.) The incompleteness of green does not make the color green linguistic, but, if anything, only makes green logically capable of being referred to. I hold that it is only with respect to its functioning as ontological "*glue*" that the incompleteness of incomplete senses and of functions is essentially the *same*, except that if anything, metaphors like unsaturatedness or incompleteness seem *more* suited to illuminate the incompleteness of senses than the incompleteness of functions. Being glue is not the same as being linguistic, certainly not intensionally speaking. There is at the very least a distinction in reason here. I would say that glue is the logically prior aspect of incompleteness, since it is shared by functions and incomplete senses alike, and is prelinguistic as such. I want to say that glue is the genus and linguistic nature is the difference defining the species of incompleteness of incomplete senses. I admit, as a weakness in my view, that the incompleteness of a sense only makes it logically capable of being expressed by a name, much as the incompleteness of a function only makes it logically capable of being referred to by a name. One might therefore object that the sense of "green" is not being made linguistic here any more than the color green is. It does not even help me here that for Frege, the color green is an extensional in sense (2) mapping function, while the sense expressed by "green" is intensional in sense (2), i.e., is one of many ways a thing can be presented. Being intensional in this sense does not make a thing linguistic. For modes of presentation are intensional in this sense too, and

they are cognitive as opposed to linguistic. Thus I can only appeal to the (I think strong) intuition that the sense of "green" is linguistic and the reference of "green" — the color green — is not, conjoined with the fact that the mode of presentation is purely cognitive as such (Frege appears to keep its traditional role in what little he says about it), and thus cannot be what makes the sense linguistic. But my argument is an inference to the best explanation; in fact, I think, to the only explanation: Why is Frege so concerned to distinguish a sense from the mode of presentation it contains, if the reason is not that senses are linguistic and modes of presentation are cognitive?

Dummett expounds on how we must introduce special quantifiers for senses, since they are not objects or functions. But we do not need new quantifiers for senses, or for that matter, forces or tones. I explained how all these can be accommodated by systematic reference shifts in appropriate sentential contexts. This is not only logically feasible, but Fregean as well. Since Frege expressly admits such shifts for senses, yet introduces no special quantifiers for them, charity suggests Frege is well aware that special quantifiers for senses are not needed. For we can simply tell from what (would otherwise be) the referentially opaque context that senses are being quantified over[5]. And I argued that forces and tones are best understood as broadly belonging to indirect speech as well. Nor do we need special quantifiers for ideas, insofar as Frege says they can be "taken as" objects. But if ideas cannot be taken as objects, then we cannot quantify over them at all, certainly not in a public notation.

Dummett finds no private language arguments in Frege, and is puzzled why Frege appeals to a permutation problem instead of to the Julius Caesar problem to show that courses-of-values have not yet been fixed. Thus it is doubly curious why Dummett thinks permutations only concern the objective-subjective distinction. For on that very view, the permutation argument in *Grundlagen* can only be a private language argument. And Dummett is by implication disagreeing with Quine's argument in "Ontological Relativity" that permutation problems continue in the objective realm (Quine [1971] p. 27; see p. 28-35). Thus in order to show that Frege should have relied on the Julius Caesar problem instead, it is not enough to claim that permutations concern only the objective-subjective distinction, at least not for Quine or me, since for us the claim is not true. Quine's solution to Frege's permutation problem as it occurs in the objective realm would be that we can refer

[5] Compare Frege's peer, Hugh MacColl, who simply leaves the detection of shifts from material mode to formal mode and back in his symbolism up to the reader. Like Frege's shifts to indirect reference, MacColl's shifts are dependent on the context, but unlike Frege's, they are unsystematic.

to courses-of-values, whatever *they* are, in our home language. Frege's solution is to regiment all names into logical determinacy, in effect fixing the identity of every reference through the nonidentity of discernibles, and also by making every identity statement determinate. In any case, the problem of permutations is a general one for the whole realm of objective reference, not just courses-of-values. Perhaps this point is obscured by the fact that except for truth-values, courses-of-values *are* the only objects Frege expressly admits in his formal notation. Perhaps the point escapes Frege as well or is not important to him, since he discusses only the permutation of courses-of-values in *Grundgesetze*. But Quine is clear that any such permutations merely illustrate the general problem, since he offers so many different examples (*ibidem*).

Dummett says Frege's *Grundgesetze* thesis that sentences are "singular terms....appears to provide a conclusive reason why Frege could no longer maintain the context principle as one governing *Bedeutung* (Dummett [1995] p. 10). But the thesis appears as early as *Begriffsschrift*, and is thus apparently held by Frege from *Begriffsschrif*t through the expressly contextualist *Grundlagen* to *Grundgesetze*, and is thus apparently seen by Frege as being consistent with his contextualism. I think the thesis is not mentioned in *Grundlagen* only because it is unimportant to the *Grundlagen* project of providing a popular exposition of Frege's logicist analysis of numbers as complex logical objects. Frege says in *Begriffsschrift*:

> To be sure, one can distinguish between subject and predicate here, too, if one wishes to do so, but the subject contains the whole [propositional] content, and the predicate serves only to turn the content into a judgment. *Such a language would have only a single predicate for all judgments, namely, "is a fact"*. We see that there cannot be any question here of subject and predicate in the ordinary sense. *Our ideography is a language of this sort, and in it the sign* ⊢ *is the common predicate for all judgments* (Frege, *Begriffsschrift* § 3, Frege's emphasis).

Thus sentences are singular terms as early as *Begriffsschrift*, and quite compatibly with the ordinary sort of subject-predicate distinction we make within a sentence. We need only add that singular terms are names, so as to infer from this that Frege's unprefixed sentences are names. But there is a problem. Frege distinguishes assertion from mere supposition in *Begriffsschrift* § 2, so that even the predicate "is asserted" or the prefix "I hereby assert that" cannot assert. Now, Frege admits that his predicate, "is a fact," is not a "predicate in the ordinary sense." But that "the predicate serves only to turn the content into a judgment" blurs his assertion-supposition distinction completely. For his common predicate is the vertical judgment stroke plus the horizontal content stroke. Thus the judgment stroke,

which is what indicates assertion, is a logical part of the predicate. This is a blunder because a predicate is a logical part of what is asserted, and the indicator of assertion cannot be a logical part, nor even a logical part of a logical part, of what is asserted.

The new thing in *Grundgesetze* is not the thesis that sentences are names, but the nature of the vertical stroke. Sentences remain names, but the vertical stroke is now only deemed an assertion operator, as opposed to being also deemed a logical part of the predicate. Thus if there remains a subject-predicate distinction at all, it can only be a distinction in reason within the unprefixed sentence. Thus the *Begriffsschrift* blur of predication and assertion is removed. The context principle, of course, applies *because* of assertability; but it applies only *within* a sentence, i.e., within a sentence which may be either asserted (prefixed by the vertical stroke) or merely supposed (not so prefixed). That sentences now refer to truth-values may be another new thing, but is irrelevant.

My three suggested methods for avoiding paradoxes like Russell's are methods of prevention in general, not of discovery in particular. Keith Donnellan's [1972] distinction between referential and attributive use of definite descriptions may help here. I am saying attributively (and hypothetically) that if any expression, whatever it may be, is paradoxical, then it will be neutralized in the way the method prescribes. Thus we need not actually single out which expressions are paradoxical in order to do arithmetic safely. To borrow a phrase from Russell, this may seem to have "all the advantage of theft over honest toil." But to reply with a pragmatic phrase, "Work smart, not hard." Why not here? Granted, the price we pay for saving arithmetic so easily is that foundational understanding of the sort only discovery of problems can give is not involved. But we can always pursue that too. The threat of hidden paradox can be easily avoided, but it is far from boring. My only point is that arithmetic is not, as Frege thinks, tottering. For we can easily save it from whatever threat of paradox there may be.

Dummett says he also takes the forward road in intuitionism. He says,

> Intuitionists do not think [that a mathematical] statement acquires a meaning only when it is proved or disproved. How could anybody set about trying to prove or disprove a conjecture if he did not know what it meant? Rather, for intuitionists, one understands a mathematical statement if one is able to recognise a proof or disproof of it: its meaning consists in an effective classification of mathematical constructions into those that are proofs of it and those that are not (Dummett [2007] p. 126).

But how can we know what a conjecture means if we do not understand it? Surely to know what a conjecture means *is* to understand it; or at least these are logically equivalent. And if we know what a conjecture means before we provide any actual proofs or disproofs, then we know what classical mathematics means after all. What, then, is wrong with classical mathematics? That was my original criticism.

Dummett says, "From an intuitionistic standpoint,...an understanding of a mathematical statement consists in the capacity to recognize a proof of it when presented with one; and the truth of such a statement can consist only in the existence of such a proof" (Dummett [2000], p. 4; [1977] p. 6; [1974] p. 8). But as I noted before, "a proof" is an existential generalization; and Dummett says the intuitionist always requires an instance. Thus for the intuitionist, we do not understand a mathematical statement until a specific proof or disproof is actually found. And that raises the Meno problem—unless, of course, we can know what a mathematical statement means without understanding it.

What is the capacity to recognize a proof of statement S? In the broadest sense, it is the capacity to recognize that an argument for S is valid and has true premisses. In this broad sense, all of classical mathematics is intuitionistically meaningful. To be a significant intuitionist, Dummett must have in mind some more restrictive kind of recognition, but without begging the question against classical mathematics. Would he say that we are unable to recognize a proof if and only if there is no proof to recognize? In a trivial sense, that version of the backward road is true; but is it not also true that we *would* be able to recognize one if, perhaps *per impossibile*, there *were* one? Or would he say that we do not know if we are able to recognize a proof if we have not found one yet? That too makes proof prior to understanding a mathematical statement, and I think also prior to knowing what it means.

To sum up, Dummett and I basically agree that the road in the cognitive order is forward from sense to reference in Frege, though a few disagreements remain on related issues. I thank him for his very helpful clarifications, and I will be delighted if my response is helpful as well.

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