Twenty Fregean Ways to Quantify Over Frege's Senses

– Jan Dejnožka –

Abstract: This paper continues my discussion with Michael Dummett on Frege's senses, published in *The Philosophy of Michael Dummett*[1] and further developed in *Diametros*.[2] In his reply to my original paper, Dummett came to agree with me that senses are neither objects nor functions, since they have a categorially different kind of linguistico-metaphysical function to perform. He then asks how we might quantify over senses, if they are neither objects nor functions. He discusses two main options, and finds one unviable and the other "very un-Fregean."[3] I then offer a Fregean or neo-Fregean option in my rejoinder.[4] And I still hold that this way out will do the job, or is at least plausible enough that the burden of persuasion is on those who disagree. But I hope to show in this paper that on a more complete examination of Frege, there are at least twenty Fregean or neo-Fregean ways out, with the one I proposed being option (17).

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1. An introduction to Frege's senses

What are Frege's senses? I briefly offer my own interpretation without arguing for it here. Senses are the connotative meanings as opposed to the denotative / referential meanings of sentences and terms. Complex senses, including but not limited to those expressed by sentences, can be sliced or carved up by the intellect in different ways. Frege calls the senses expressed by sentences "thoughts." Frege's term is misleading because his thoughts are not mental or private to a mind. Instead, all senses, including thoughts, are timeless, immaterial, abstract entities that can be grasped by many minds. They make public communication possible across both minds and times.[5] Thus "proposition" would be a better term than "thought." ("Thought" is not Frege's only odd term; he also calls universal properties "concepts," perhaps because they can be grasped by many minds.

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[1] Dummett (2007); Dejnožka (2007).
[2] Dejnožka (2010).
[3] Dummett (2007): 124.
[4] Dejnožka (2010): 127.
[5] Frege offers at least twelve private language arguments, Dejnožka (1996/2003): 109, 289 n.1.

But as usual, I shall use Frege's own terms when discussing him.) The same thought can be expressed in assertions, questions, and commands, as in "The cat is on the mat," "Is the cat on the mat?," and "Put the cat on the mat."[6] For the later Frege, statements express thoughts, but do not describe facts. Instead, they attributively denote, and also cognitively single out and refer to, truth-values. For Frege, truth-values are abstract objects which he calls "the True" and "the False," and every statement both denotes and refers to exactly one of them. This is Frege's semantic version of the law of excluded middle. All senses both denote and refer to the referent, if any, of linguistic expressions that express them.

Senses explain the possibility of informative identity and existence statements. An informative identity statement is an identity statement whose subject-terms express different senses as opposed to being mere labels, and an informative existence statement is an existence statement whose subject-term expresses a sense as opposed to being a mere label.

Senses are the indirect referents of indirect speech (oratio obliqua). Indirect speech is speech about speech, including quoted speech. Senses are also the indirect referents in cases of what Quine calls referential opacity and Russell calls propositional attitude. For example, "I believe that the cat is on the mat" expresses my belief that a certain thought (in the realm of sense) is true, i.e., denotes or refers to the True, as opposed to my belief that a certain fact (in the realm of reference) is the case. Unlike Russell and the early Wittgenstein, Frege never admits facts as a metaphysical category, though as far as I can see, there is no good reason why he could not. It would be less elegant, but it would be more complete. And as we shall see, Frege's metaphysics is already quite redundant as it is.

Frege says the senses expressed by object-names (including sentences as names of truth-values) are "saturated" or "complete," and the senses expressed by predicate-names are "unsaturated" or "incomplete." That is because he thinks of predicate-names as logically formed by removing a name from a sentence and replacing it with a variable. Almost no one would agree today, but that is his view.

Dummett and I have agreed all along that senses are essentially ways that things can be presented, as opposed to things that are presented. We agreed all along that this is their logico-linguistic function. We even agreed all along that Frege bases metaphysical category on logico-linguistic function. But for some reason or reasons, Dummett did not put two and two together and see, until he read me, that these facts imply that Frege's senses cannot be Frege's objects or, for that matter, Frege's functions. For their logico-linguistic function is essentially different, and so their metaphysical categories are essentially different. Senses are ways of presenting things, while objects and functions are things that are presented. Senses can be presented as indirect referents by higher-level senses, while objects and functions are level 0 referents that can be presented, but cannot present anything else in turn.

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6 *Pace* Frege (1918/1968): 512.

I have argued many times that senses are not objects.[7] I just stated my basic logico-linguistic argument in the previous paragraph, but in this paper I shall omit the detailed textual argument that goes with it.[8]

In his reply to my paper, Dummett repeats my logico-linguistic argument in his own way.[9] He also more or less notes my textual argument that when carefully examined, all the relevant Frege's texts support the view that senses are not objects, and no relevant text goes against it.[10] Dummett does not agree with every textual interpretation I give. But in any case, Dummett very kindly says:

> Thus, I recant my earlier view and am now in full agreement with Jan Dejnožka that senses—even thoughts—cannot be objects. He deserves credit for perceiving this.... The whole apparatus of objects, concepts, and functions is inapplicable in the realm of sense. Dr. Dejnožka perceives this too.... I think now that Frege ought to have held that view, and I applaud Dr. Dejnožka's recognition of this.[11]

But Frege is well aware that his senses are not his objects, and expressly says as much. Frege says in "On Sense and Reference," "A truth-value cannot be a part of a thought any more than, say, the Sun can, for it is not a sense but an object."[12] Dummett is unsure about Frege's level of awareness even in that quotation, due to some beautiful theoretical considerations.[13] But it is a fact that Frege says right in his major paper on senses that a truth-value "is not a sense but an object." Frege is telling us in the plainest terms that a truth-value is not a sense *because* it is an object. And as the saying goes, "There is nothing like a sordid fact to slay a beautiful theory."

2. Dummett's two main options for quantifying over senses

Dummett rather oddly limits his discussion to thoughts. Dummett finds that if thoughts are not Fregean objects, then there are only two main options: either thoughts are not objects at all, or they are a special sort of objects. He finds problems either way, but finds the first option unviable, and resolves the main problem he finds with the second option.

Dummett asks:

> What follows from a denial that thoughts are objects? Certainly that they neither fall within the domain of ordinary first-order bound variables nor lie within the range of application of ordinary first-level predicates. Do they then form a domain of quantification of their own, and can special predicates be defined over this domain? This is to regard them as objects of a different *sort* from physical objects, numbers, and

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[7] Dejnožka (2010): 118–119; (2007): 81–95; (1996/2003): 68; (1981): 36; (1979): 51.
[8] Dejnožka (2007): 81–95.
[9] Dummett (2007): 122.
[10] Ibidem.
[11] Ibidem: 122–123.
[12] Frege (1892a/1970): 64, my emphasis, quoted in Dejnožka (2010): 119; (2007): 89.
[13] Dummett (2007): 123–124.

the like. It is a very un-Fregean idea: he never countenanced distinct sorts of objects with distinct domains of quantification. If he had, then surely he would have taken numbers and human beings to be of different sorts...[14]

As Dummett is well aware, Frege does admit essentially different sorts of objects. He admits concrete (causal) objects such as human beings and abstract (noncausal) objects such as numbers and the axis of the earth. But since they are all objects, they all fall under the same first-order existential quantifier and universal quantifier. Thus Dummett is right that Frege "never countenanced distinct sorts of *objects* with distinct domains of quantification." But precisely because human beings and numbers alike are *objects*, Dummett's argument from numbers and human beings to objects and thoughts is a non sequitur. For the distinction between concrete objects and abstract objects is far shallower than the difference between objects and thoughts. And the difference between referents (including objects) and senses (including thoughts) is deeper and more general yet. Thus, it is simply a non sequitur to suppose that if Frege admitted a distinct domain of quantification for thoughts (or more generally for all complete senses), he would have admitted distinct domains of quantification for concrete objects and abstract objects, which Frege holds are both objects. Crucially, Dummett overlooks that Frege does admit distinct sorts of *entities* with distinct domains of quantification. Namely, objects and *functions* have different quantifiers and are distinct domains of quantification for Frege. For functions and their quantifiers are always higher-level than objects and their quantifiers.[15] Admittedly, those are the only two different domains of quantification that Frege admits. (Actually, there is implicitly an infinite series of higher-level domains of functions, since all functions need to be quantifiable over.) But it does leave the logical door open for additional domains of quantification, such as one over complete senses, and another over incomplete senses, mirroring those over complete objects and over incomplete functions. It even leaves the door open for quantification over forces, such as assertion, question, and command, and over tones of emotive expression, such as joy or horror. Frege discusses forces and tones very little, and never discusses quantification over them. But he clearly admits them. Dummett takes a reductive approach to forces and tones. But Frege says the assertion sign (judgment stroke) cannot function as a name, on pain of reducing assertions to suppositions.[16] In ordinary language, a period at the end of a sentence is normally taken to be the indicator of an assertion or judgment; but no one would think that a period expresses a sense or refers to a referent. And if it did, then the sentence would not yet be asserted. Likewise, to name an emotion is not yet to emote (convey) it.[17] Thus there is much to question about how many different categorial domains of quantification Frege would admit, if asked. And surely, he would want to say that there exist at least three forces and at least five tones, and that they have intrinsic predicable features. The question whether thoughts are a special kind of objects is just

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[14] Ibidem: 124, Dummett's emphasis.
[15] Frege (1893/1967): §§ 21–25; see Furth (1967): xxx–xxxv.
[16] Frege (1893/1967): §5; see (1918/1968): 513–514.
[17] For more on forces and tones as unique categories, see Dejnožka (2007): 93–97; (1996/2003): 69–70, 120, 237; (1982): 13–14.

the tip of the iceberg.[18] But the present paper suggests many other options. In the case of forces and tones, I must leave it to the reader to list the options; there should be twenty each, mutatis mutandis.

To sum up this section so far, if thoughts can be a special sort of objects, the question that arises is that if Frege expressly uses the same quantifiers for concrete objects and abstract objects, both of which are merely different sub-kinds of objects, but expressly admits different quantifiers for the deeper and more generally different domains of objects and of functions, where functions are not a kind of objects at all, then by parity of reason, should he not also admit different quantifiers for the even deeper and more generally different domains of referents and of senses, and perhaps even for forces and for tones, if they cannot be reduced to any of Frege's other categories?

3. Dummett's dilemma

The main problem Dummett finds with the option that thoughts are a special sort of objects is that John Myhill formulates a "paradox, modelled on Russell's, concerning thoughts."[19] It is simply a version of Russell's paradox in which thoughts instead of classes lead to the paradox. Dummett says Myhill's paradox "at first sight provides us with a reason for rejecting the view of thoughts as a separate sort of objects,"[20] much as Russell's paradox at first sight makes classes look problematic.

The main problem Dummett finds with the other main option, that thoughts are not objects at all, is this:

> One solution is to deny that thoughts are objects, even objects of a different sort from physical objects and numbers, for the statement of the paradox required only quantification over thoughts and over concepts under which thoughts fall. What is the price of this denial? If thoughts are not objects of any sort, they cannot be quantified over; nor can we define predicates over them.[21]

I find this to be another one of Dummett's non sequiturs on the face of it. Dummett overlooks that if senses are not objects, then quantification over senses would simply be a kind of indirect speech. And everyone agrees that for Frege, in indirect speech, senses are indirect referents. And we need not even reach that solution. For Frege, quantification is always second-level. For Frege, to say "Dogs exist" is not to say of dogs that they exist, but to predicate the second-level existential quantifier of the concept *dog*. Likewise, to say "Thoughts exist" is not to say of thoughts that they exist, but to predicate the second-level existential quantifier of the concept *thought*. Why then is it necessary for, or even relevant to, quantification that thoughts be objects?

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[18] I argue that talk of senses, forces, and tones alike is best regarded as so many kinds of indirect speech in Dejnožka (2007): 93–97.
[19] Dummett (2007): 124.
[20] Ibidem.
[21] Ibidem.

In any case, Dummett then argues that we can define at least some predicates over thoughts, therefore we can quantify over them after all, therefore they must be some sort of objects after all. Dummett says, in immediate continuation of the previous quote:

> Is this not contrary to reason? Can we not say of certain thoughts that they are malicious, of others that they are kindly, and of yet others that they are brilliant? [No, t]hese are not really predicates of thoughts, but of mental acts. It is malicious to entertain particular thoughts, kindly to dwell on others: the thoughts themselves do not have these characters, but the acts of thinking them.
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> Other predications cannot be similarly dismissed, however; it would be wrong to contend that one can say nothing about thoughts. We can attribute to them features arising from what is intrinsic to them.... Our denial to them of the status of objects can be understood not as ruling out quantifying over them but rather as excluding them from the domain of objects of the usual kinds, concrete and abstract.[22]

But this is just another one of Dummett's non sequiturs, or perhaps better, a continuation of his previous non sequitur. For on the face of it, unless we understand objects in some wide sense that includes anything that can be predicated over, it may follow, from the fact that there are predications that can be made about thoughts (and more generally senses), that we can therefore quantify over thoughts (and more generally senses); but it does not follow that thoughts (or more generally senses) are objects. For if senses are not objects, then predications about senses are just indirect speech with senses as indirect referents. And what is wrong with that? Senses are already indirect referents in all other forms of indirect speech. And let us not have the cart pull the dog. Indirect speech is indirect speech only because senses are indirect referents, that is, only because senses are ways of presenting things. They can be directly grasped when we think, but only as ways of presenting things. Thus, all thinking is indirect presentation. This includes predicational and quantificational thoughts concerning dogs and senses alike.

In any case, Dummett concludes:

> Thoughts, then, though they are not objects in the ordinary sense, are objects of a special sort, to which we can refer and over which we can quantify within a vocabulary tailored to them and not to ordinary objects.[23]

And that is perfectly fine, if and only if we understand objects in some wide sense that includes anything that can be predicated over and thereby quantified over. But if we do not, then Dummett's non sequitur is showing.

Dummett then agrees with William Demopoulos that Myhill's paradox can be resolved as easily as Russell's paradox can, and by basically the same sort of type-hierarchy that Russell uses. Thus the Myhill paradox turns out to be a paper tiger, and the dilemma a false dilemma after all. Dummett says, "This is a clumsy necessity, needed

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to resolve a tiresome paradox."[24] But it is no necessity. In fact, it is well known that a type-hierarchy is overkill and rules out infinitely many innocent expressions as ill-formed. I explain my own far simpler and less drastic way out of Russell's paradox, local self-destruction of any paradoxical expressions, and by parity of reason, out of Myhill's paradox as well.[25] There are other resolutions of Russell's paradox too, most famously ZFC (Zermelo-Fraenkel axiomatic set theory with the axiom of choice).

Dummett then says, as his final word on his two main options:

> To say that thoughts are objects of a special sort does not mean merely that a different range of predicates can intelligibly be applied to them, as we normally take different predicates to be applicable to human beings and to numbers: it means that they are not full-fledged objects, even though we can intelligibly quantify over them.... What disqualifies thoughts from being full-fledged objects is that they are *not self-subsistent:* a thought could not exist unless at least one human being or other rational creature grasped it. Thoughts, therefore, do not form, as Frege may well have supposed, a determinate domain containing every thought that ever will or ever could be grasped or expressed.... The domain of thoughts is indeterminate and constantly expanding. Quantification over it, therefore, cannot be explained in the classical manner as the outcome of scrutinising each element of the domain to determine whether it satisfies the predicate or not. It can be explained only in the intuitionist fashion, under which an existential statement is justified only by the production of an instance, and a universal one by a demonstration of its necessitation from the very notion of a thought.[26]

Here Dummett is changing the subject from Frege's technical use of "thought" to a very ordinary use of "thought" as meaning something that is actually grasped by a mind (but not necessarily something that is mental, though that would naturally fit in). Dummett is expressly disavowing what he says "Frege may well have supposed." I now wish to argue that for Frege, all senses, including all thoughts, are self-subsistent in the sense that they are timeless abstract entities that logically cannot fail to exist, and moreover, that Frege is right about that, certainly within his own metaphysics. I shall give two arguments for this, one based on the timelessness of truth, and the other on the timelessness of concepts.

My first argument is that truth and falsehood are timeless, thoughts are true or false, and all senses are either thoughts or logical components of thoughts, therefore all senses are timeless. And if senses are timeless, then they logically need never be thought of by anyone.

Dummett overlooks in his "Reply" that the timelessness of thoughts is precisely how Frege explains how indexical sentences appear to change truth-value across persons, times, and places. Namely, if I falsely said yesterday in Michigan, "I am here now in Iowa," and if you truly say today in Iowa, "I am here now in Iowa," there is no real change in truth-value because we used the same sentence to express different timeless

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[24] Ibidem.
[25] Dejnožka (2007): 103–104.
[26] Dummett (2007): 125–126, my emphasis.

thoughts, and my thought is false while yours is true. Frege is very clear about this in many texts, most famously in "The Thought."[27]

My second argument assumes some controversial metaphysics. It assumes that properties (Frege's concepts) are ante rem universals. The argument is that properties are ante rem, every sense is essentially based on some property that is a way a thing can be presented, therefore senses are essentially ante rem, and therefore are timeless.

Now, one might object that this is a non sequitur of my own, and that it does not follow from the fact that senses are essentially based on timeless properties that senses are themselves timeless, and therefore need not be thought of in order to exist. For a tree or rock can essentially have timeless properties, such as that of being a concrete object, but trees and rocks are logically contingent. My reply is that "based on" and "have" are not the same. Also, my argument is not based merely on properties' being timeless, but on their specifically being ante rem. And my conclusion was not merely that senses are timeless, but that they are specifically ante rem. And that means they can exist even if they have no instances, in the sense that they need not be senses "of"[28] any denotations / referents. And that entails that they need not be thought of, since (or in the sense that) they need not be instances of what anyone is thinking of. In contrast, trees and rocks are in re, and logically cannot have instances.

Dummett overlooks in his "Reply" that Frege's incomplete senses are timeless because Frege's concepts (properties) are timeless. Specifically, he overlooks that incomplete senses are ante rem universals because Frege's concepts are ante rem universals. Concepts (recall that for Frege concepts are properties of things, not mental ideas in the mind) are universals because different objects fall under literally and numerically one and the same concept. Different horses fall under literally one and the same concept *horse*. They are ante rem universals because some concepts have *no* objects that fall under them. There are no unicorns to fall under the concept *unicorn*. Even more decisively, some objects *logically cannot* have any objects fall under them. Frege gives as examples "our old friends the square circle and wooden iron."[29] To see this, we need only recall that the predicates "horse," "unicorn," "square circle," and "wooden iron" not only denote / refer to literally the same *concept* across any objects they apply to, but they also express literally the same *sense* across any objects they apply to. And if the concept is ante rem, then any incomplete sense which essentially uses it as a way of presenting a thing can only be ante rem as well. For all incomplete senses are senses "of" (in Furth's sense) timeless concepts that are their essential conceptual content. The sense expressed by "is a three sided closed plane figure" is precisely the sense "of" (in Furth's sense) the concept *three sided closed plane figure*. Likewise for the sense expressed by "is a dog" and the concept *dog*. And it is logically necessary that a predicate that expresses a sense applies to an object if and only if that object falls under the concept that the sense is a sense "of," that is, if and only if that concept maps the object onto the True. For that concept is precisely the concept that the predicate refers to. It is that predicate's referent. And for Frege, neither the sense nor the concept it is a sense "of" can change across any logically

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27 Frege (1918/1968): 516–518.
28 In Montgomery Furth's helpful sense of "of," Furth (1967): xix.
29 Frege (1884/1974): § 74.

possible persons who may grasp them, nor across any logically possible times at which they may be grasped. Thus, they can both only be ante rem universals. I would say they are distinct only in reason, and differ only in their logico-linguistic function. And if incomplete senses must be ante rem, then by parity of reason, i.e., for categorial reasons, all complete senses, including thoughts, must be ante rem too. In fact, every thought must include an incomplete, i.e., predicative, sense. However, it does not follow from the fact that the incomplete sense it includes is ante rem that the thought itself is ante rem. For that would commit the fallacy of composition. Hence, we must rely on parity of reason, here categorial reason. All this is part of what I call Frege's hall of mirrors.

Thus, all senses are ante rem, and are in that sense self-subsistent. But what disqualifies senses from being referents in the ordinary sense is not that they are not self-subsistent, *even if Dummett is right that they are not self-subsistent*. It is instead that senses are essentially ways of presenting things, and ordinary referents are essentially presented things that can present nothing further in turn. The logico-linguistic functions are too deeply different for senses to be level 0 referents, meaning referents that are not ways of presenting lower level referents. In fact, the logico-linguistic functions are formally contradictory. For a thing cannot both be a way of presenting something else and not be a way of presenting something else. Compare particulars as ultimate logical subjects of predication that cannot be predicated in turn of still lower level logical subjects. Particulars logically cannot be both ultimate logical subjects and logically predicable of something. And there is a parallel vicious infinite regress argument in each case. If everything is a way of presenting something else, then there will be an infinite series of presentations of presentations, and nothing will ever be presented. And if everything is a property of something else, then there will be an infinite series of properties of properties, and nothing will ever have a property. The regresses are parallel precisely because an incomplete sense (way of presenting something) is always a sense "of" a concept (property), and a predicate expressing the sense applies to an object if and only if the object falls under the concept. I accept both regress arguments, but here I am concerned only with the illumination their similarity brings, and not with whether they are sound. In fact, in light of the essential connection every sense "of" a referent (in Furth's sense of "of") has to some concept the referent falls under, this too is part of Frege's hall of mirrors. But it is not wholly redundant, since senses are intensional and concepts are extensional. Note that it is logically implicit in Frege that there are infinitely many higher levels of senses and of concepts. For there logically can always be higher levels of indirect speech about indirect speech, and higher levels of quantifier-concepts predicated of concepts. Likewise, more generally, for functions. Concepts are a kind of functions; concepts are functions that map truth-values onto their arguments.

Here my own qualified objects are an alternative to Frege's senses. Qualified objects are timeless ante rem objectual ways that things logically can be presented. But that is another story.[30]

The old adage is, when faced with an apparent contradiction, draw a distinction. And the truth is that the word "object" is said in many ways. For Frege, objects are

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[30] Dejnožka (1996/2003): xxvi, 47, 61, 73; (2015/2021): 30, 575–576, 590; (1987): 1–16.

referents that are particulars, i.e., ultimate logical subjects of predication, and that are not ways of presenting anything else. Following Quine, objects in the wide sense are entities, that is, anything that can be quantified over, that is, anything that belongs to some domain of quantification. The necessary and sufficient condition of this is that they can be identified and differentiated, hence counted. For counting and quantification are logically equivalent. Frege says, "Affirmation of existence is in fact nothing but denial of the number nought."[31] And of course there must be predicates. But since properties are always the basis of identifying and differentiating objects in the wide sense, the applicability of predicates is already implied. As Panayot Butchvarov says, "Absolutely bare things are absolutely incapable of identification and thus of existence."[32] Even Gustav Bergmann's bare particulars are essentially numerical individuators and essentially exemplify universals. Now, in this wide quantificational sense of "object," even Frege's functions are objects. For they can be quantified over. Why not then also his senses, forces, and tones? For some, this wide sense of "object" applies to any object of perception or thought, including Meinong's golden mountain and round square. But Frege is no Meinongian. Like Russell and Quine, Frege holds that there is no such thing as a merely possible object, let alone an impossible object. Frege says, "A merely possible figure is no figure at all."[33]

If we admit senses, we do want to quantify over them. We want to be able to say that there are senses, that senses exist. We also want to be able to say that two terms express two senses, and so on. And we want to be able to say things about senses, such as that they are ways of presenting things. And we can do all that in ordinary language. But are there any viable formal ways to quantify over senses, perhaps even Fregean or neo-Fregean ways? This is important to Frege scholarship and philosophy alike. For if Dummett and I are right that senses are not objects in Frege's sense, surely Frege would want, and should at least implicitly already have, some way he could quantify over his own senses!

4. Twenty Fregean ways to quantify over Frege's senses

Dummett's discussion of two ways out strikes me as far too general. I think it will be more productive to look instead at the specific mechanics of Frege's categories, and at what specific adaptations of them might work. So to speak, I propose to look at sordid facts more than beautiful theories, and simply let Dummett's two general options fall where they may; and it may not always be easy to tell.

In my rejoinder to Dummett in *Diametros*, I explained a Fregean way to quantify over senses. Again, it is option (17) in the list of twenty options below. Unfortunately, Dummett died about a year later. I had sent him a copy of my 2010 paper, but I do not know if he was able even to read it, due to his health. I had been hoping to discuss these things with him further, and possibly to arrive at a more complete agreement on senses; but it was not to be. Except for my few published disagreements with him, I basically

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[31] Frege (1884/1974): § 53.
[32] Butchvarov (1979): 122.
[33] Frege (1895/1970): § 126.

agree with everything he says about Frege. He will always be the world's best Frege scholar to me.

But I wonder why Dummett discusses only quantification options that he finds either unviable or un-Fregean. For I can think of at least twenty Fregean options that would seem to do the job. The main requirement is that a sense needs to correspond one-one with whatever may be used to stand in for it in the quantification, so that truth will be preserved. And Frege is very big on correspondences. His metaphysics is almost like an echo chamber, or a hall of mirrors. Of course, the stand-in entity, if any, must be quantifiable over.

I proceed to describe the twenty ways.

First, every predicative sense essentially corresponds one-one with the concept (property) it is a sense "of" in Furth's sense. The predicate "x is a dog" expresses a sense that is essentially a sense "of" the concept *dog*, and so on. Now, consider Eubulides' paradox that I see the hooded man (the referent of the object-name "h"), but do not know he is my brother (the referent of the object-name "b"), even though they are the same person. (This is like Frege's example of the Morning Star and the Evening Star.) Now, we wish to assert the quantificational statement, "There are three senses expressed in the statement 'h = b'." (I am waiving further subdivisions of the thought, or propositional sense, expressed by that statement.) Surely the sense expressed by "h" can be represented by the referent of the concept-name "x is the hooded man," the sense expressed by "b" can be represented by the referent of the concept-name "x is my brother," and the sense expressed by the identity sign can be represented by the referent of the relational concept-name "x = y".

This works because a sense is a way of presenting something, and a way of presenting something must have a basis in a property that the presented thing has, or would have if the thing existed. (In Frege's ideal language, all presentation is veridical.) And every predicative sense corresponds one-one with the concept it is a sense "of." More generally, every function-name expresses a sense "of" some function. (Again, concepts are a kind of functions.) Object-names express a sense "of" an object (if any), but since an object can only be identified by a property it has, some sense "of" that property is implicitly part of the sense "of" the object. For example, the sense of "x is my brother" is implicitly part of the sense expressed by "b". (This is an ordinary language example. In the ideal language, the term "my" would not occur, since it is indexical.)

Thus, this one-one correspondence applies to all senses. For all senses are either senses "of" a concept or senses that implicitly include the sense "of" a concept. And a sense would not be the sense it is unless it had the basis in a concept that it has. That is the attributive-denotative side of Frege's semantics, in both ordinary language and the ideal language.

This sort of quantification over senses may be called representational. For the analogy is precisely to Frege's representation function, which is well-defined in his formal notation, and which makes functions correspond one-one with, and mutually represent, their courses-of values. And if functions can represent their own courses-of--values (and vice versa) in quantification, then why cannot functions (including concepts) also represent in quantification the senses "of" which they are the basis, in a different

but analogous sense of "represent"? And there is no doubt that for Frege, all functions (including concepts) can be quantified over.

Second, since functions, including concepts, correspond one-one with their courses-of values, we can also use the courses-of-values to represent the sense expressed by each function-name, in yet another analogous sense of "represent." Since functions and their courses-of-values are distinct only in reason (they are interdefinable), options (1) and (2) are distinct only in reason as well; but they are different for that very reason. And for Frege, courses-of-values can be quantified over, since they are objects.

There is no Russellian "backward road" problem for options (1) and (2) in the ideal language, where every entity has one name that expresses one sense. Even in ordinary language, the sense "of" the complete essential concept of a thing will correspond one-one with the thing.

Third, we can take Frege's names as representing their own senses in representational quantification. For there is an essential one-one correspondence of the sense with a certain name (either type or token) that expresses it, even if many names (physical marks that express senses) can express the same sense. For if a name expresses a different sense, then it is not the same name. And for Frege, names can be quantified over, since they are objects, even if the senses they express are not.

This option fits nicely with my interpretation of Frege's identity statements as asserting the relation that the two subject-names refer to the same referent, and as being factually informative if and only if the two subject-names express different senses.[34] For in "entity if and only if identity" ontology, we would ideally like identity statements and quantifications to be about the same entities, in this case names.

Of course, the relation of a name to its sense is many-one in ordinary language, where we can introduce indefinitely many different names (types or tokens) that express the same sense. And the relation of names expressing different senses to the object (if any) which is the referent of all the names is many-one there as well. But an ideal language would not be so redundant.

This option also ties quantification to a notation, i.e., to language. For names are part of language. But I think that Frege would not mind that. For Frege doubts that humans can grasp senses, including thoughts, "without the garb of language."

Fourth, a sense can be represented by the mode of presentation it contains. Frege always takes great care to distinguish senses from the modes of presentation he says they contain. Thus on my interpretation of Frege, a mode of presentation is not identical with the sense that contains it, even though both are equally intensional.[35] And this implies that there is a categorially necessary one-one correspondence between any sense and the mode of presentation it contains. For the mode of presentation is essentially unique to the sense that contains it. The sense is not even identifiable as the sense it is, except in virtue of the mode of presentation it contains. (Complex senses contain correspondin-

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[34] Dejnožka (1981).
[35] Dejnožka (2010): 121, 126–127. On the face of it, senses are logico-linguistic in function, while the modes of presentation they contain are their cognitive element. Differences among perceptual modes of presentation would ground and explain pre-linguistic and even pre-human informative identity judgments for Frege.

gly complex modes of presentation. For example, there are three modes of presentation contained in the thought that h = b.) Modes of presentation are intensional for the same reason senses are. Namely, both sorts of entity function as ways of presenting things, not as presented things. In any case, we can identify and distinguish modes of presentation, make essential predications about them, and quantify over them in all the same ways that we can quantify over senses, though of course senses and modes of presentation would stand in for each other, not for themselves.

Options (1)–(4) are various kinds of representational quantification over senses. I proceed to describe four corresponding options of another kind.

Here we analogize quantification over senses to Frege's suggestion that a mental idea can be "taken as" an object, as opposed to analogizing quantification over senses to his representation function. Of course, temporal private mental ideas and timeless public objective senses are mutually exclusive categories. But that does not preclude the possibility that senses can be 'taken as' the objects or functions they are senses "of," more or less as ideas can be 'taken as' the objects they appear to be. No backward road problem will arise in the ideal language, nor in ordinary language for the sense "of" the complete essential concept of a thing.

Representation and 'taking as' are not at all the same relation. For they are deeply, categorially different in their relata. Representation is between functions and their co-urses-of-values. 'Taking as' is between objects and mental ideas. Functions are timeless, abstract (noncausal) ante rem universals, and are publicly cognizable. (All functions are universals because they are literally the same entities across mappings. They are ante rem because some, such as *unicorn* and *round square,* have no entities falling under them.) Mental ideas are temporal, concrete (causal) particulars, and are private to a single mind. Also, nothing can fall under or within an idea, since ideas are particulars, i.e., ultimate logical subjects.

All this duplicates our first four options as follows.

Fifth, we can quantify over senses by 'taking them as' the referents of the same function-names we used to represent senses in option (1).

Sixth, we can quantify over senses by 'taking them as' the courses-of-values we used to represent senses in option (2).

Seventh, we can quantify over senses by 'taking them as' the names we used to represent senses in option (3).

Eighth, we can quantify over senses by 'taking them as' the modes of presentation we used to represent senses in option (4).

Options (5)–(8) may be called kinds of 'taking as' quantification over senses.

We can mix and match options (1)–(8) to derive more options. For on Frege's acceptance of the principle of the identity of indiscernibles, every entity has a unique set of properties. Now, all those unique sets of properties can be used both in representational quantification over senses and in 'taking as' quantification over senses. This doubles our options from eight to sixteen. This is our second doubling of options. Note that a thing's unique set of properties need not be limited to its unique complete essence.

All sixteen options are very Fregean indeed. What could be more Fregean than Frege's representation function or his 'taking as'? In fact, these two notions are uniqu-

ely Fregean. Who else had them before Frege? No one, since one would have to admit Frege's unique categories first. Of course, earlier thinkers could have developed similar notions based on similar categories. The whole history of metaphysics is something of a hall of mirrors.

We can and must introduce a generic determinable here. I take entities' *standing in* for senses in quantification to be a determinable of which there logically can be indefinitely many determinates. I have found sixteen determinates. Eight fall under one proper sub-determinable, kinds of representation, and eight fall under another, kinds of 'taking as'.

There are also four more options, the first two of which we have already discussed.

Option (17) is to regard quantification over senses as a kind of indirect speech.[36] The argument would be: senses can only be talked about in indirect speech, senses can be quantified over, therefore quantification over senses is indirect speech.[37]

Option (18) is more strictly correct. Fregean quantification is always higher-level than the things we ordinarily think we are quantifying over. For Frege, "Dogs exist" is not about dogs. It is the predication of the second-level quantifier-concept *does not have nothing falling under it* of the first-level concept *dog*. Likewise for quantification over senses. On every type-level of senses, "Senses exist" is not about senses. It is about the appropriate level-concept *sense*. Frege has several explicit or implicit arguments for this concerning objects,[38] and all the arguments would apply to senses.

Option (19) is this. Frege says there are "two wholly different cases [where] we speak of existence. . . . In the one case the question is whether a proper name designates, names, something; in the other, whether a concept takes objects under itself. "[39] Thus, instead of using second-level quantifiers, "Dogs exist" can be treated as simply meaning "The term 'dog' designates (denotes / refers)." Likewise, "Senses exist" can be treated as meaning "The term 'sense' expresses."

Option (20) is this. Frege says: In order to speak of the sense of an expression 'A' one may simply use the phrase 'the sense of the expression "A"'.[40] Thus the phrase is clearly an exception to Frege's rule that expressions beginning with the singular definite article refer to objects. He is treating the phrase as indirect speech precisely because senses are indirect referents.[41] And from there we could quantify over senses. The phrase "the concept *c*" is Frege's other main exception to the rule. That phrase designates an object that represents the concept.[42] For the later Frege, this object is the concept's course-of-values. I think "the force F" and "the tone T" ought to be exceptions to the rule too.

Option (20) supports option (17), but option (18) is still more correct for quantification.

All twenty options are very Fregean. May others find more.

Dummett's first option is that senses are not objects at all. If so, he finds them unquantifiable simpliciter. But I have shown twenty ways to do it, including simpliciter

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[36] This is the option I presented in Dejnožka (2010): 127.
[37] See Dejnožka (2007): 81–97 for more on indirect speech.
[38] Dejnožka (1996/2003): 75–76; (1979): 10–18.
[39] Frege (1895/1970): 104.
[40] Frege (1892a/1970): 59.
[41] Ibidem: 59, 65.
[42] Frege (1892/1970): 46–47.

as option (17). His second option is that senses are a special kind of objects. That is viable too. My own qualified objects do everything senses do. But I agree with him that "It is a very un-Fregean idea."[43]

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[43] Dummett (2007): 124.